<u>Using *Sun Moon Planets Simulation* and *Minor Planet Comet Search Utility*</u>

Preface:
The *Sun Moon Planets Simulation* and the accompanying *Search Utility* are provided free for use by anyone with an interest in astronomy. It is based on the following: *Astronomical Algorithms* by Jean Meeus, *Astronomy on the Personal Computer* by Oliver Montenbruck and Thomas Pfleger, *VSOP87* by P. Bretagnon and G. Francou, *SPA* by the National Renewable Energy Laboratory, *ELP-2000/82* by M. Chapront and J. Chapront, and *Fundamentals of Celestial Mechanics* by J.M.A. Danby.

Introduction:

After having taught solar system astronomy for many years, I decided to develop a *Windows* based simulation software called *Sun Moon Planets Simulation*. This program is intended for educational use and amateur solar system observations.

Sun Moon Planets Simulation is a free program that uses astronomical algorithms to solve basic equations in celestial mechanics. The program simulates the orbital motion of the planets (Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune) around the Sun. The planetary motion is solved for any selected day(s) and then the data is put into a set of tables for observational applications.

The tables of solar system data include:

- Sun - Rise, Transit, Set Times; Ephemeris; Solar Eclipses
- Moon - Rise, Transit, Set Times; Ephemeris; Moon Phases; Lunar Eclipses
- Planets - Rise, Transit, Set Times; Ephemeris; Phenomena; Equinox and Solstice Dates

Contents:

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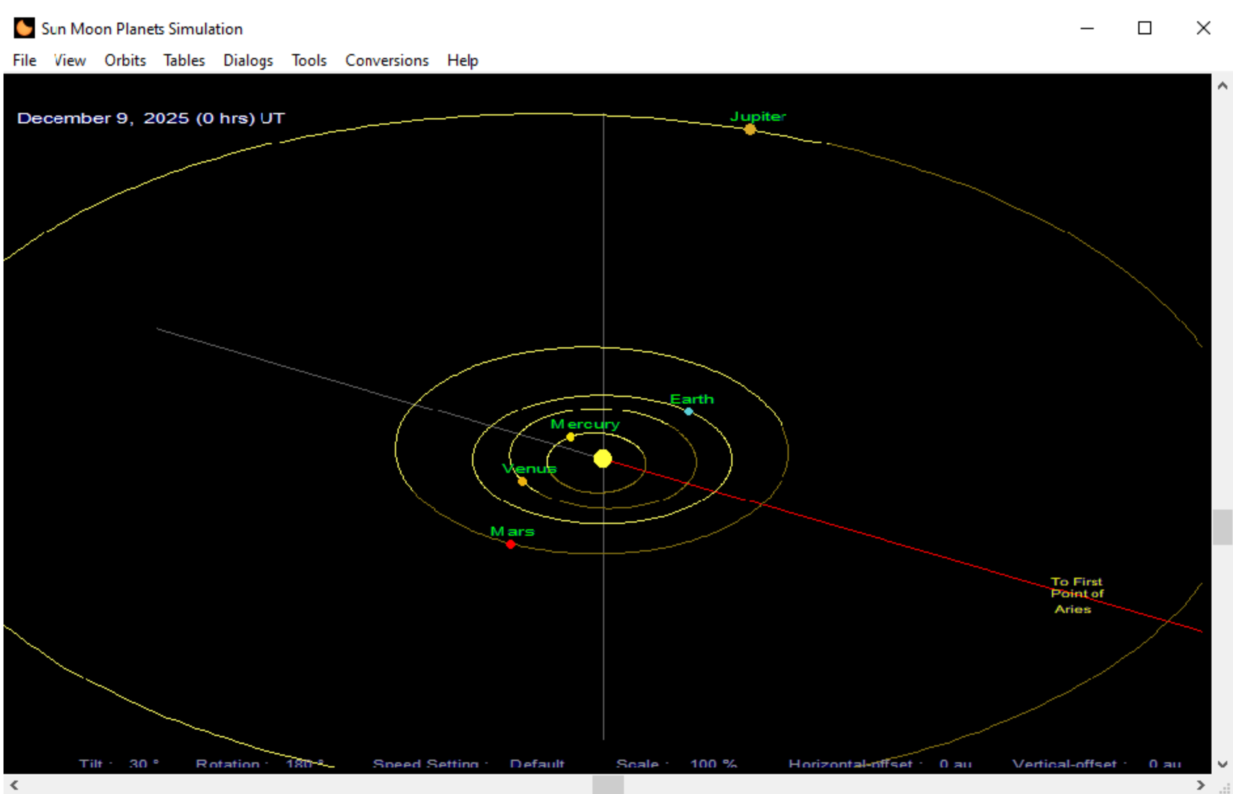
1: A Quick Overview of SMP Simulation



The opening screen of the program displays the daily motions of the planets as they orbit around the Sun. Take note of the menu items: **File**, **View**, **Orbits**, **Tables**, **Dialogs**, **Tools**, **Conversions** and **Help**. This brief overview will discuss each menu item.

The program also provides a set of informative tables regarding the Sun, Moon, and planets.

Sun Tables include:

- Sun data such as its distance, equatorial and ecliptic coordinates.
- Sun rise, transit and set times.
- Sun ephemeris.
- Solar eclipse tables.

Moon Tables include:

- Moon data such as its distance, equatorial and ecliptic coordinates.
- Moon rise, transit and set times.
- Moon ephemeris.
- Daily Moon phase angles.
- Quarterly phases of the Moon.
- Lunar eclipse tables.

Planet Tables include (for each major planet):

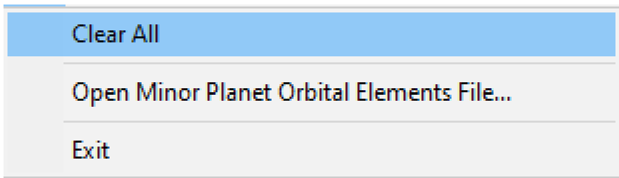
- Planet data such as its distance, equatorial and ecliptic coordinates.
- Planet rise, transit and set times.
- Planet ephemerides.
- Planetary phenomena.
- Earth equinox and solstice dates.

Each of the tables allow you to enter a date (between 1900 and 2100) to determine information about the items listed above.

The program also provides a set of tools to change the date or to animate the motion of the planets as they orbit the Sun.

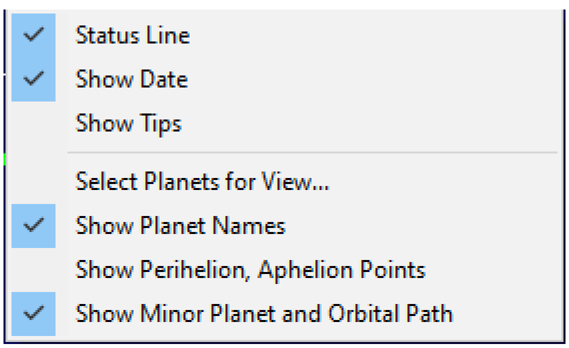
Menu Items

File



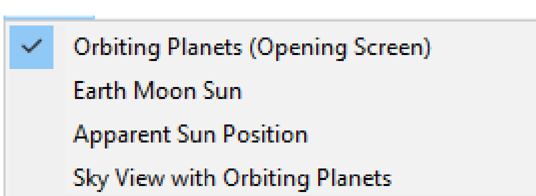
The **File** menu item allows you to open an orbital elements file of data that was obtained from the Minor Planet Center. The orbital elements file is used to plot the orbit of an asteroid or comet.

View

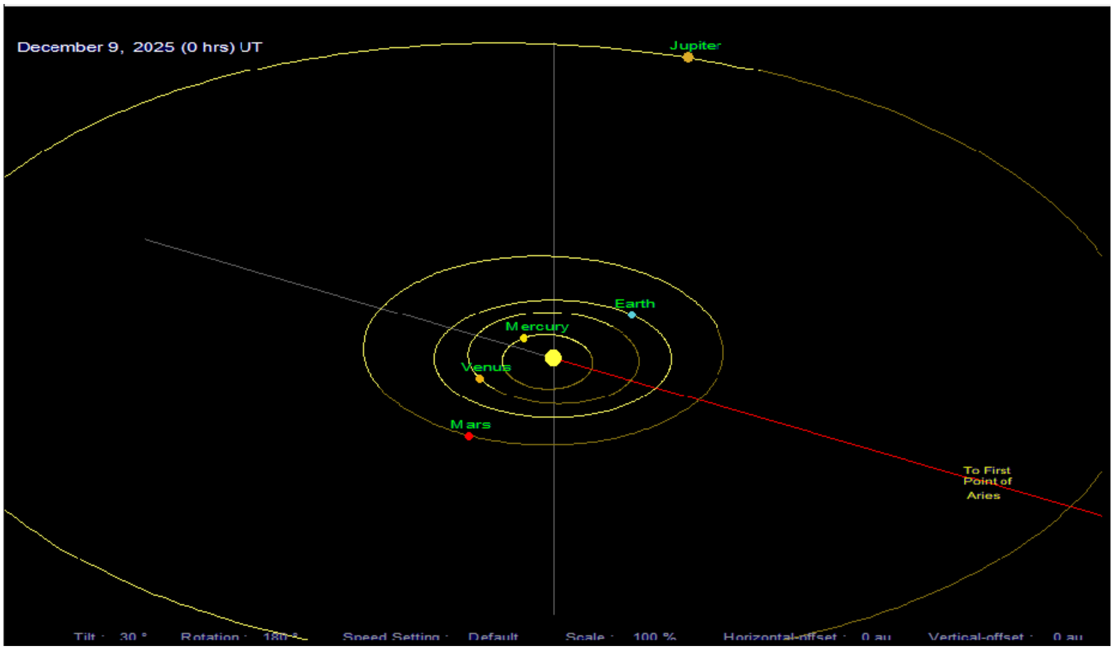


The **View** menu item allows you to turn on or off some of the features such as the **Status Line** and **Tips** regarding the planets and orbital paths.

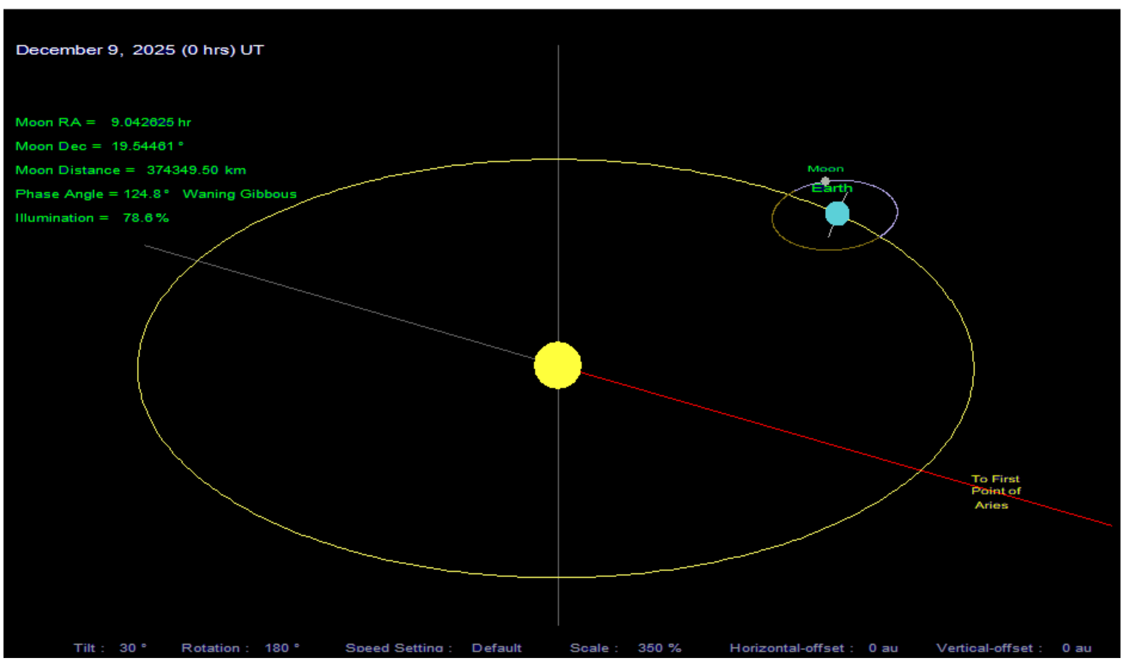
Orbits



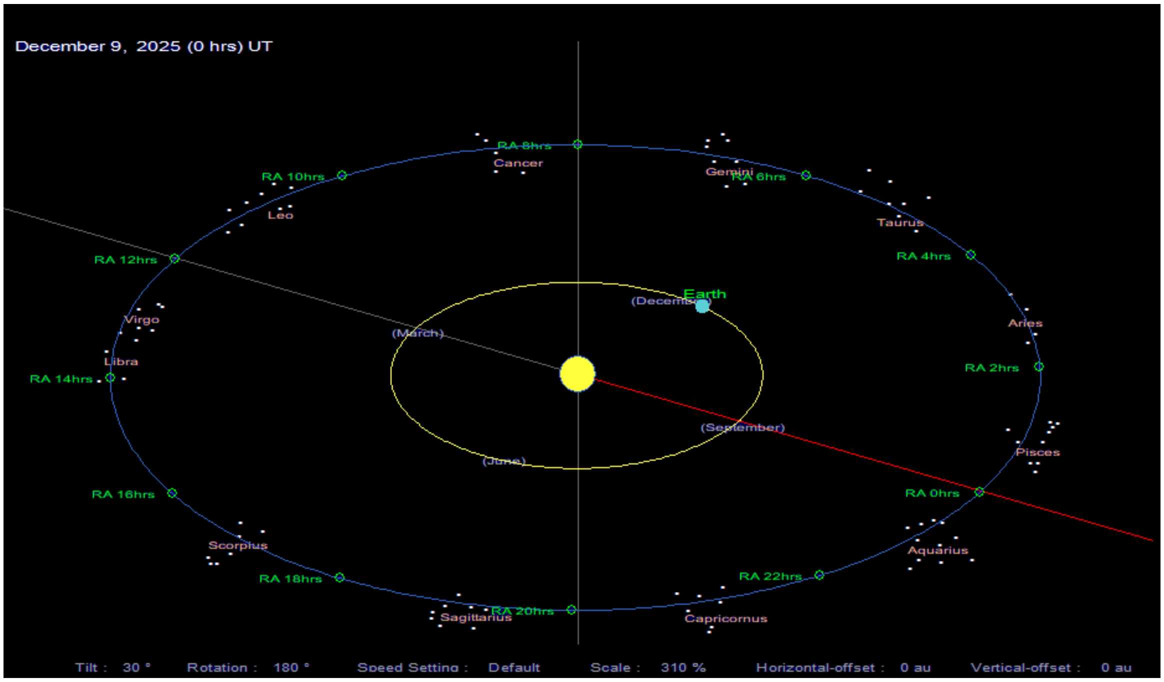
The **Orbits** menu item allows you to see the following:



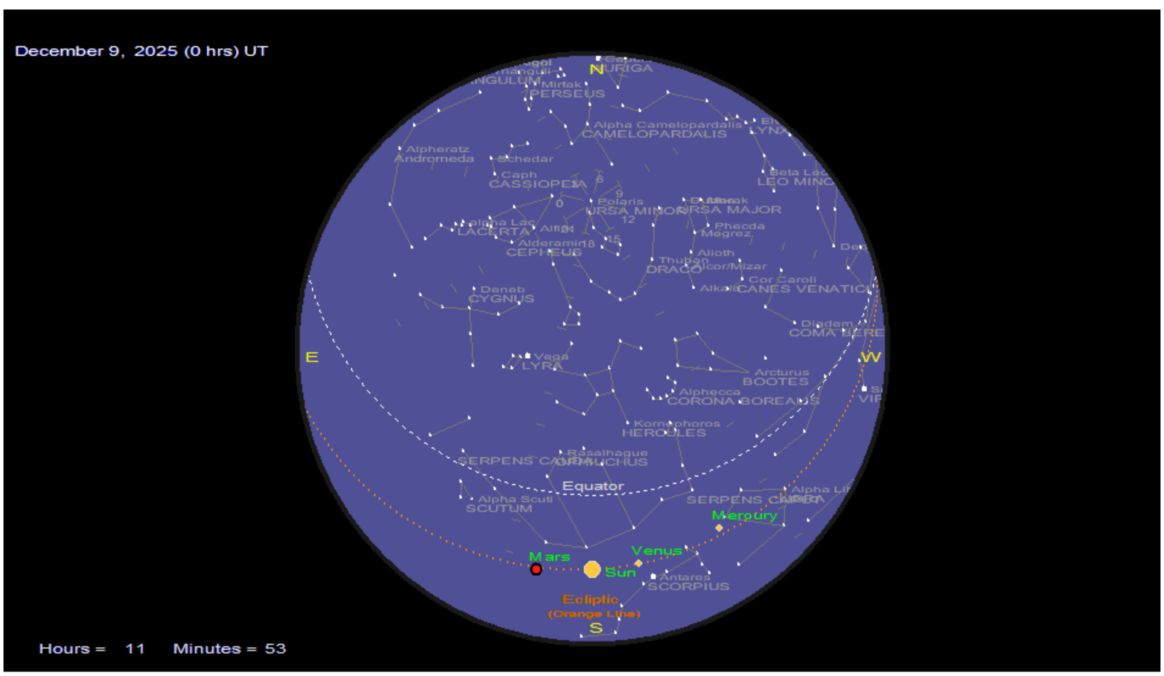
Orbiting Planets



Earth Moon Sun



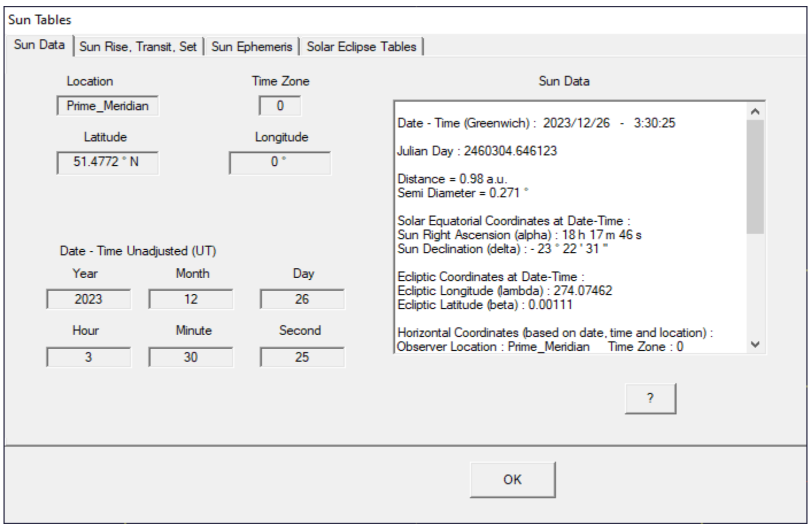
Apparent Sun Position



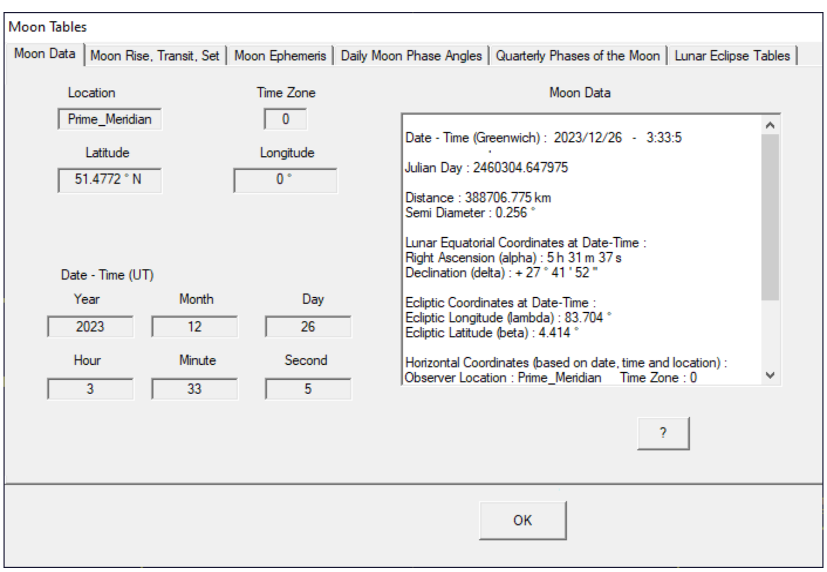
Skyview with Orbiting Planets

Tables

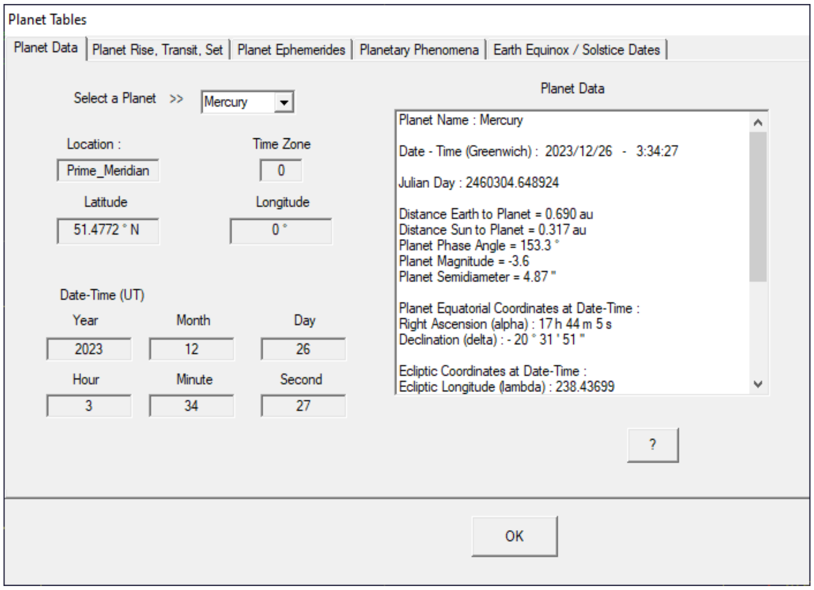
The **Tables** menu item contains: **Sun Tables, Moon Tables** and **Planet Tables**. As described above, these tables provide information regarding the Sun, Moon and planets.



Sun Tables

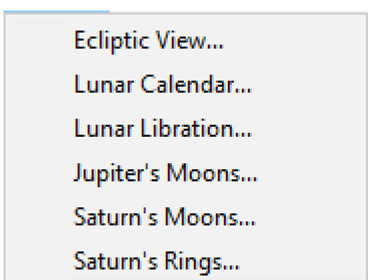


Moon Tables

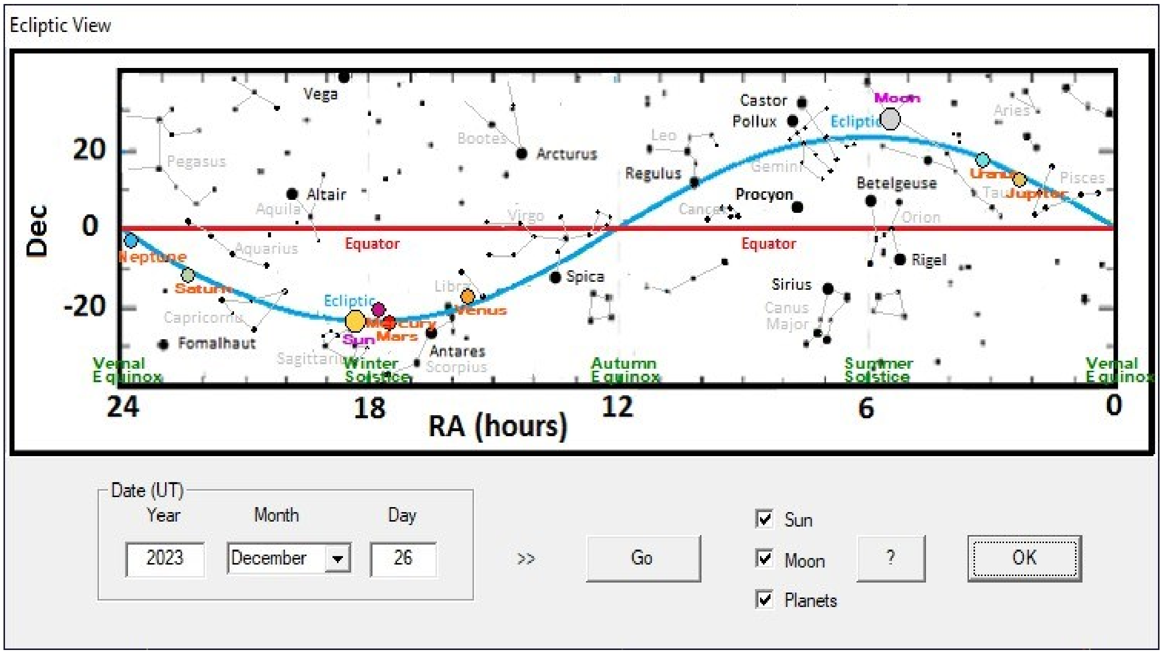


Planet Tables

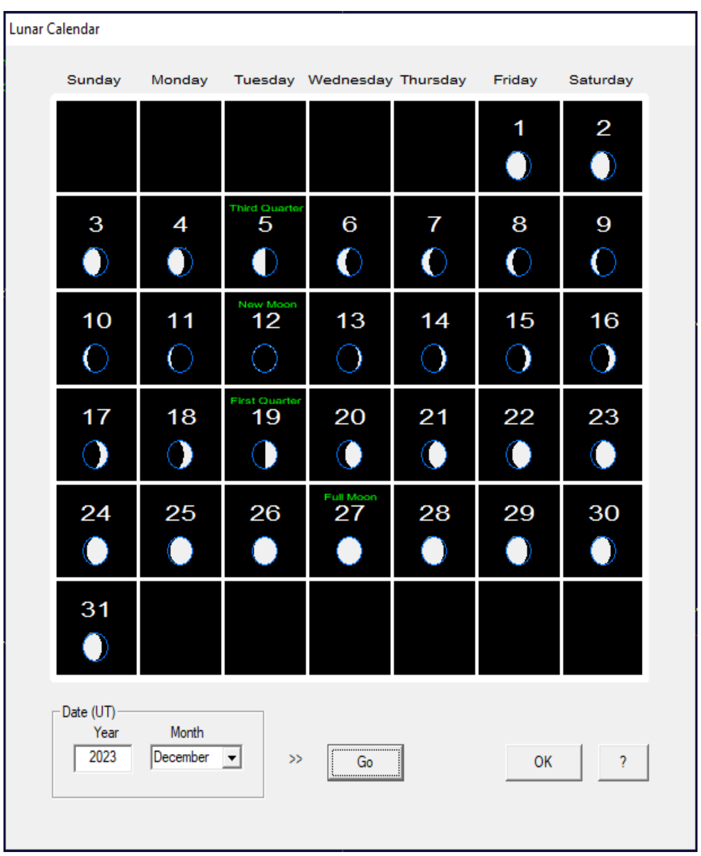
Dialogs



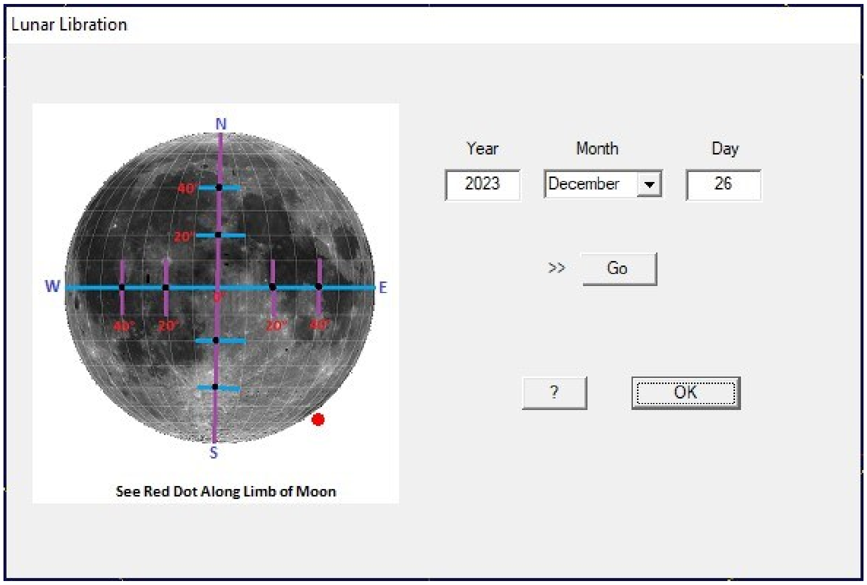
This is a collection of dialog boxes that show a particular view of some part of the Solar System. It includes an **Ecliptic View, Lunar Calendar, Lunar Libration, Jupiter's Moons, Saturn's Moons, and Saturn's Rings**.



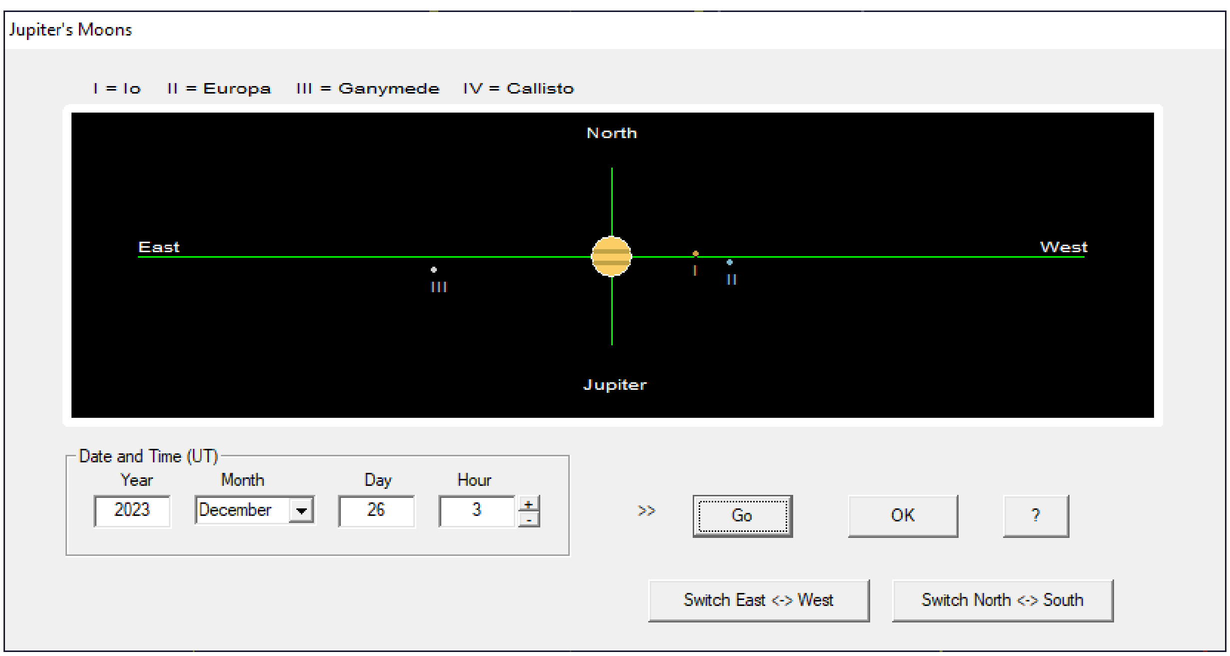
Ecliptic View



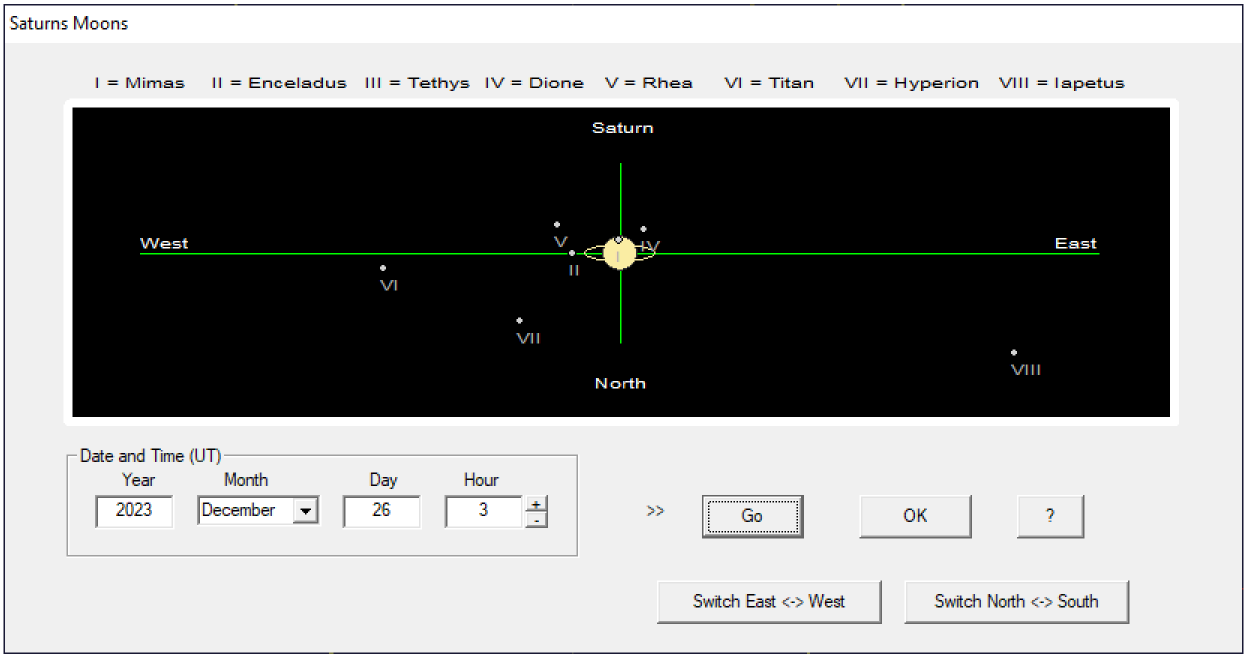
Lunar Calendar



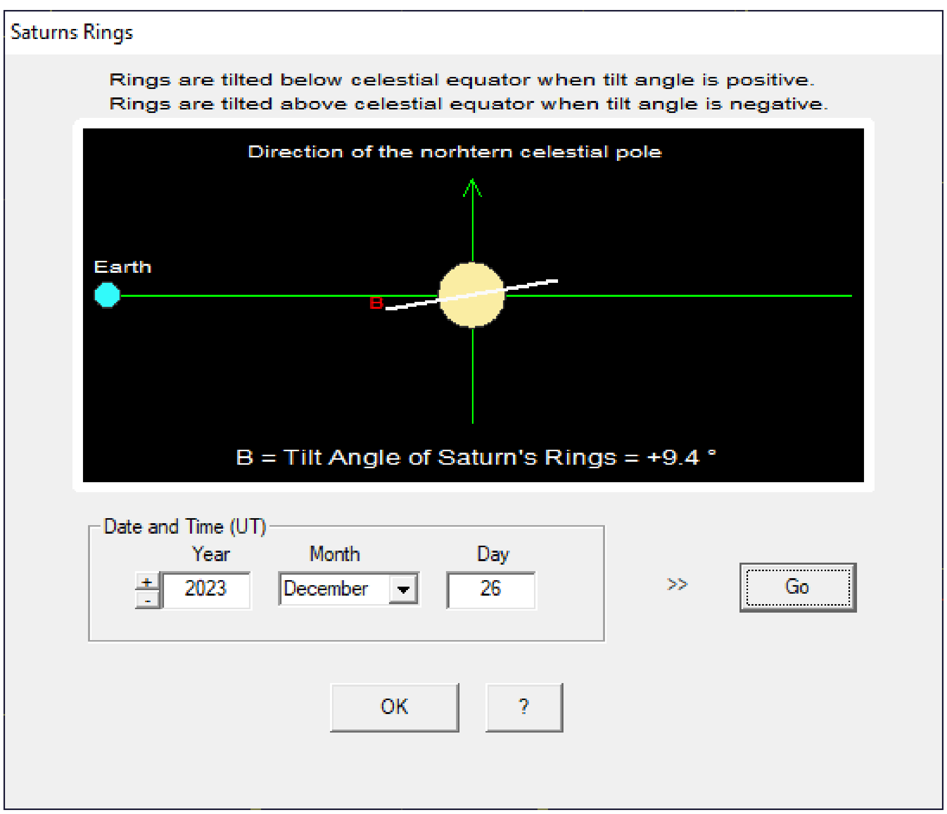
Lunar Libration



Jupiter's Moons

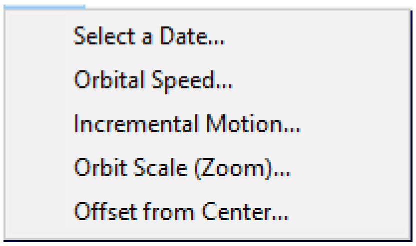


Saturn's Moons

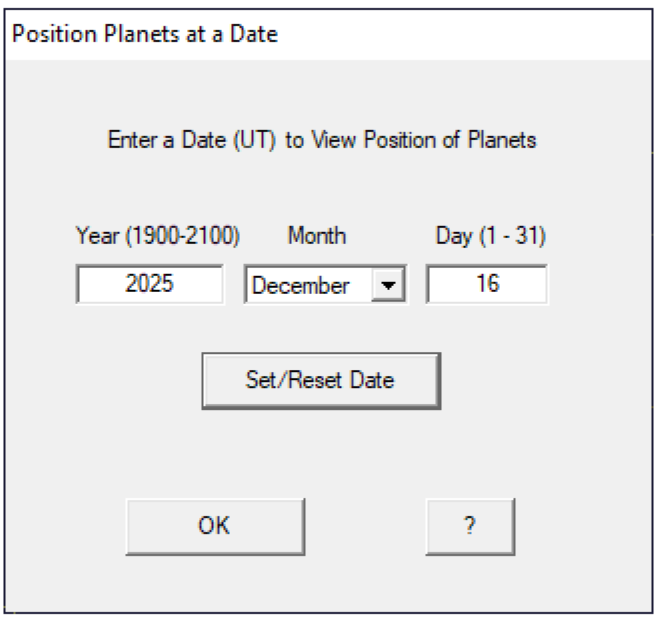


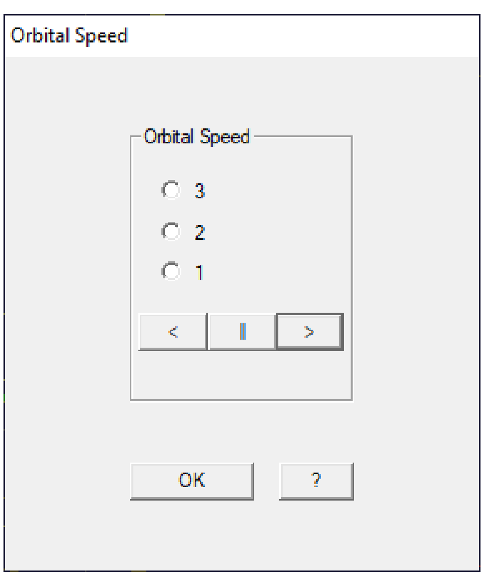
Saturn's Rings

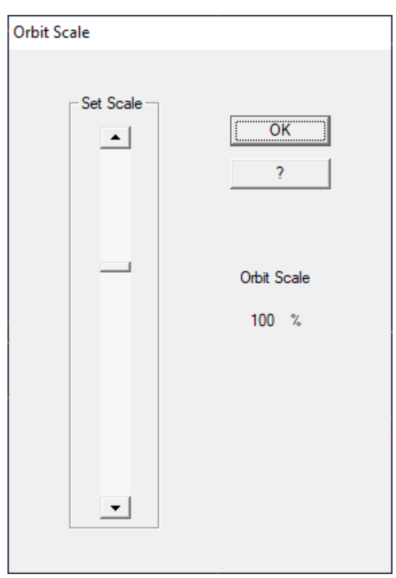
Tools



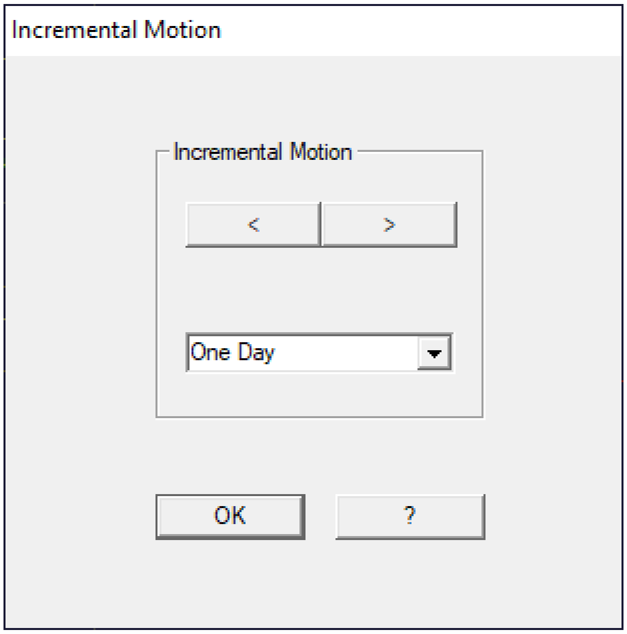
This menu item provides tools such as **Select a Date**, **Orbital Speed** that allows you to animate the main screen so that you can observe the planets orbiting the Sun. You can also set the **Orbital Scale (Zoom)**.



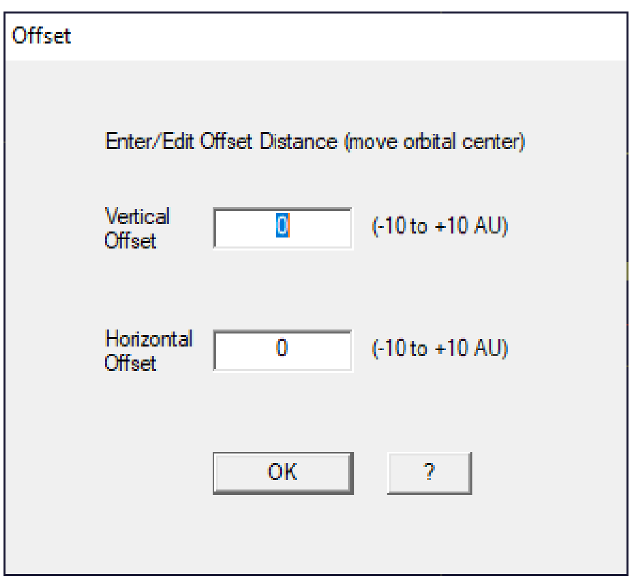




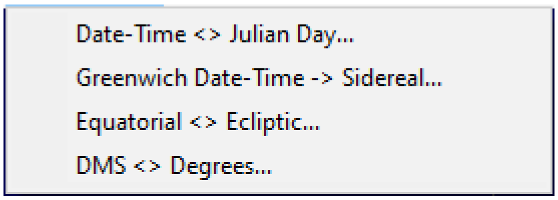
There is also a tool for **Incremental Motion** so that you can move forward or reverse in time by a day, month. 3 months, 6 months, or a year.



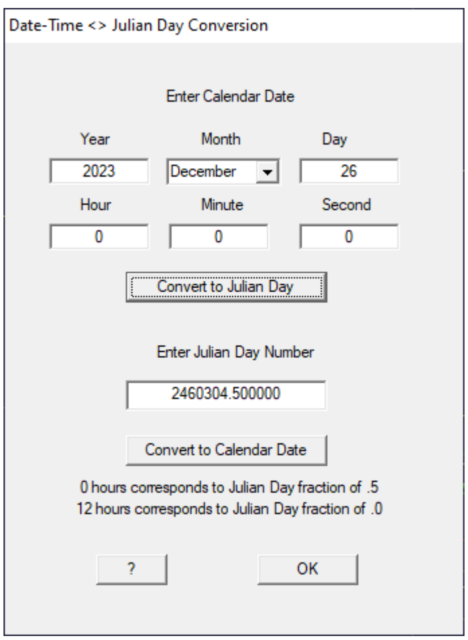
An **Offset from Center** tool allows you to shift the center of the solar system around the screen to improve the view if necessary.

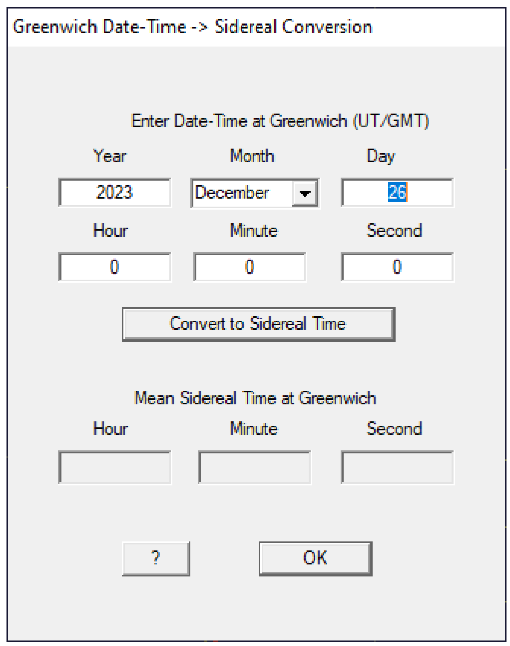


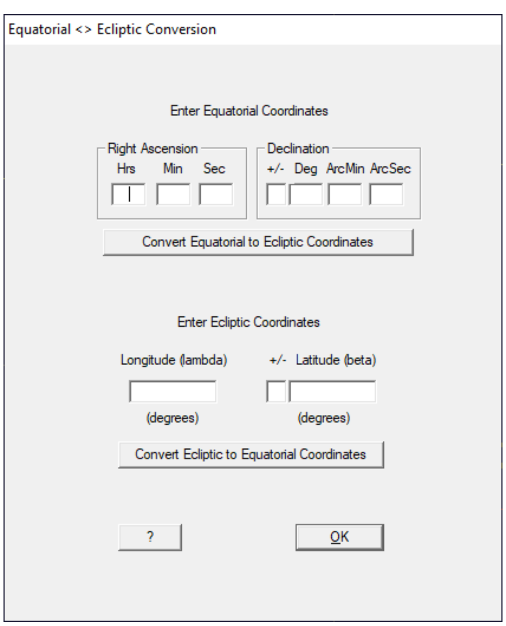
Conversions



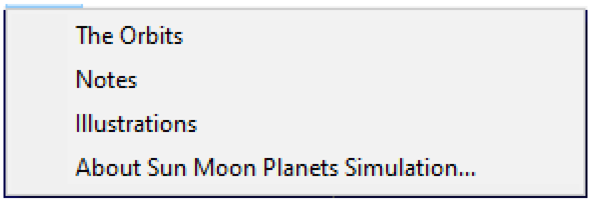
This menu item allows you to make various conversions that are helpful for the rest of the program.







Help



The **Help** menu item provides information regarding orbit views plus a set of **Notes** and **Illustrations** that explain some of the terminology.

Examples of How to Use this Program

1) The opening screen of the program shows the position of the planets at the present date (0 hours UT). Use the menu item **Select a Date** to change the year to the next year or the previous year. Again, note the position of the planets, particularly that Earth is still in the same position and Mars has moved approximately halfway around its orbital path. Try advancing the month and days and note the changing position of the planets. Approximately how long did it take for Mercury and Venus to make a complete orbit around the Sun?

Definitions:

Synodic Period - Time between two successive identical positions of a planet as seen from Earth.
Sidereal Period - True orbital period of a planet, the time required for the planet to complete one full orbit of the Sun.

2) On the opening screen adjust the vertical slider (right-hand side of screen) to about the mid-position. Note all the planets appear to be orbiting in a flat plane. This plane is called the Ecliptic. Click on **Dialogs** and select **Ecliptic View**. In the dialog box click the check box to show the planets. Note how the planets are all very close to the blue line (Ecliptic). The blue line gives the RA and Dec of any planet, Moon or the Sun at any date. Inside the dialog box adjust the date by increasing the month and clicking the **Go** button. Note the motions along the Ecliptic of the Sun, Moon and planets.

Definition:
Ecliptic - an imaginary line in the sky that marks the apparent path the Sun follows over the course of one year.

3) Click **Orbits** and select **Earth Moon Sun.** Note the position of the Moon with respect to Earth. Click on **Select a Date** again and advance the days one at a time to 7 days. How far did the Moon go around the Earth? Advance the month by one month. How far did the Earth go around the Sun? How far did the Moon go around the Earth?

4) Reset the screen to show planets orbiting the **Sun**. Set the date to March 20 (or March 21 - depending on the year).

Click the menu item **Tables** and select **Sun Tables**. Click the tab, **Sun Ephemeris** and set the date to March 20. In the first row of data (Sun Ephemeris tab) note the Date, RA and Dec. The RA of the Sun should be close to 0 (or 24) hours. Look at the relative position of the Sun with respect to the Earth (see menu item **Orbits, Apparent Sun Position**). The Earth should be directly in-line with the Sun and in opposition to the **First Point of Aries**. Next, find the date when RA of the Sun is equal to 12 hours (in September). Set that date for the **Orbits, Apparent Sun Position** and note that Earth will be in conjunction with the **First Point of Aries** and in-line with the Sun. When the RA of the Sun is equal to 24 hours (or 0 hours), Earth be back where it was on March 20.

Definitions:
Right Ascension (RA) - Right Ascension is measured starting at 0 hours when the Sun is at the **First Point of Aries**. This is where the Sun crosses the celestial equator from south to north at the Spring Equinox (also called Vernal Equinox).
Declination (Dec) - the angular distance of a point north or south of the celestial equator.

5) Click menu item **Tables** and select **Planet Tables**. In the dialog box click the tab **Earth Equinox / Solstice Dates**. Note the date of the June (Summer) Solstice. Click the screen menu item Date and change the month and day to the date shown in the Planet dialog box. On the main screen you should see the relative position of the Sun and Earth at the Summer Solstice (compared to the direction of the First Point of Aries). Repeat this procedure using the dates in the dialog box to see the position of Earth with respect to the Sun at the Winter Solstice in December.

6) Click **Orbits** and select **Earth Moon Sun**. Click menu item **Tables** and select **Moon Tables**. In the dialog box click the tab **Quarterly Phases of the Moon**. In the listing find the date of the next New Moon. Click the menu item **Tools, Select a Date** and change the year, month and day to the date shown in the dialog box. Then close the dialog boxes and observe the position of the **Earth, Moon and Sun** on the **Orbits** screen. The Moon should be in-line (conjunction) with the Sun. Keep in mind that the positions of the Earth and Moon were calculated at 0 hours UT but the time of the New Moon may be a few hours more or less than that.
Repeat this procedure to view the position of the Moon at First Quarter, Full Moon and Last Quarter.
You can also observe this using the **Ecliptic View** dialog box. By changing the dates, you can observe the position of the Moon in relation to the Sun.

7) Click **Orbits** and select **Earth Moon Sun**. Click menu item **Tables** and select **Sun Tables**. In the dialog box click the tab **Solar Eclipse Tables**. In the listing find the date of the next Total Solar Eclipse. Click the **Tools** menu item and **Select a Date** and change the year, month and day to the date shown in the **Solar Eclipse Table**. Then close the boxes and observe the position of the Earth, Moon and Sun on the **Orbits** screen. The Moon should be in-line between the Sun and Earth. You may also note in the green text on the top left side of the screen that it indicates a *New Moon* and the *Solar Eclipse* at a given time UT. Note that on the screen the Moon may not be completely in position since the position of the Earth and Moon were calculated at 0 hours UT and the solar eclipse probably occurs several hours later (solar eclipses don't occur at midnight).

8) Click **Orbits** and select **Earth Moon Sun**. Click menu item **Tables** and select **Moon Tables**. In the dialog box click the tab **Lunar Eclipse Tables**. In the listing find the date of the next Total Lunar Eclipse. Click the menu item to **Select a Date** and change the year, month and day to the date shown in the **Lunar Eclipse Table**. Then close the boxes and observe the position of the Earth, Moon and Sun on the screen. The Moon should be in opposition to the Sun. You may also note in the green text on the top left side of the screen that indicates a *Full Moon* and the *Total Lunar Eclipse* at a given time UT. Note that on the screen the Moon may not be completely in position since the position of the Earth and Moon were calculated at 0 hours UT and the lunar eclipse may occur a few hours before or after 0 hours UT (midnight).

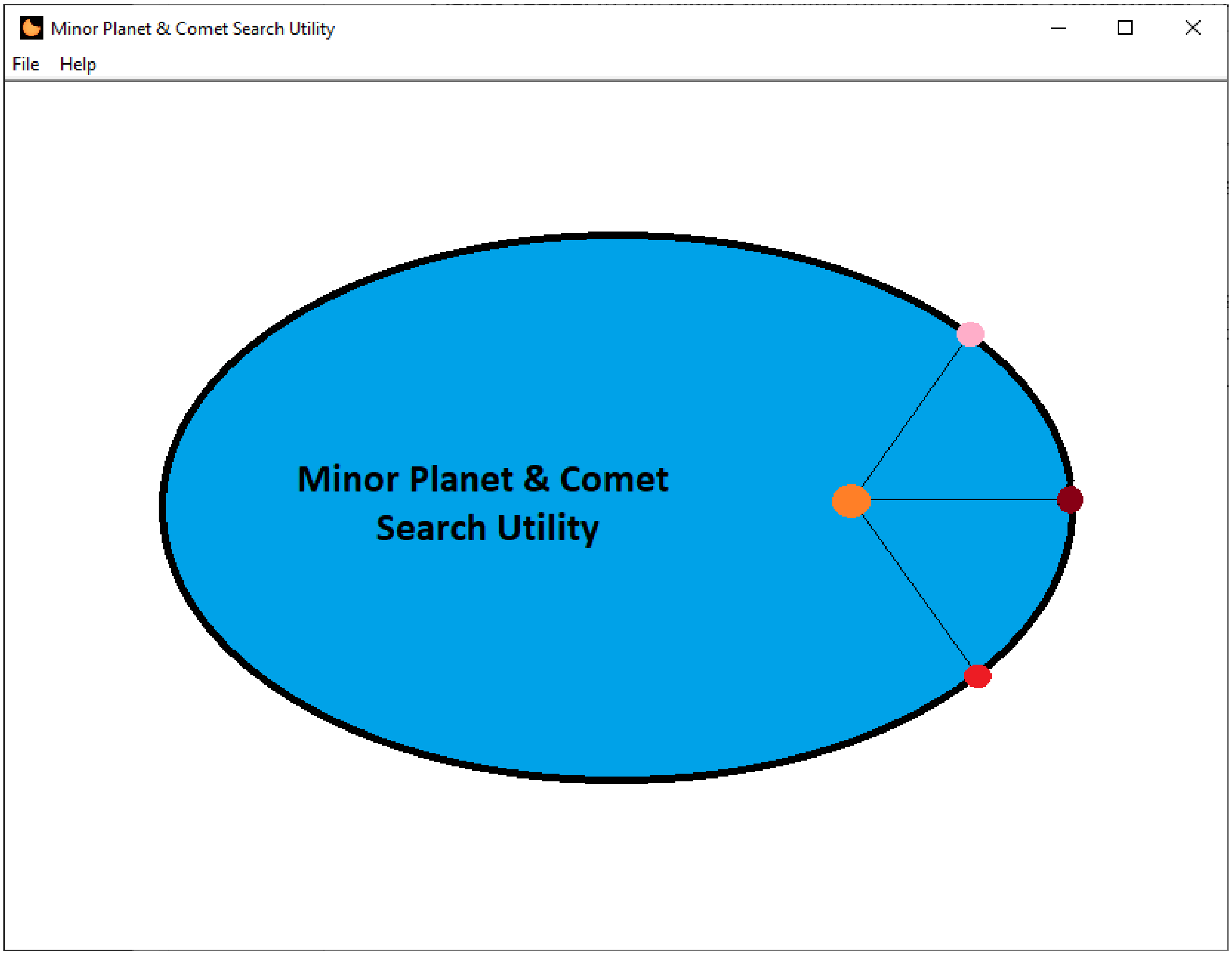
9) Click **Orbits** and select **Orbiting Planets**. You should be showing the main screen with all the planets. Click menu item **Tables** and select **Planet Tables**. In the dialog box click the tab **Planetary Phenomena**. From the combo box select the planet Mercury and click the **Go** button. In the listing find the date of the next Superior Conjunction. Click the menu item to **Select a Date** and change the year, month and day to the date shown in the dialog box. Then close the boxes and observe the position of the Earth, Mercury and the Sun. To improve the view, you might move the vertical slider to the bottom and then use the **Scale Tool** to enlarge the orbits.

10) Again, on the main screen with all the planets, click menu item **Tables** and select **Planet Tables**. In the dialog box click the tab **Planetary Phenomena**. From the combo box select the planet Mars and click the **Go** button. In the listing find the date of the next Opposition. Click the menu item **Date** and change the year, month and day to the date shown in the dialog box. Then close the dialog boxes and observe the position of the Earth, Mars, and the Sun. To improve the view, you might move the vertical slider to the bottom and then use the **Scale Tool** to enlarge the orbits. Note the green text on the top left side of the screen - *Mars at Opposition*.

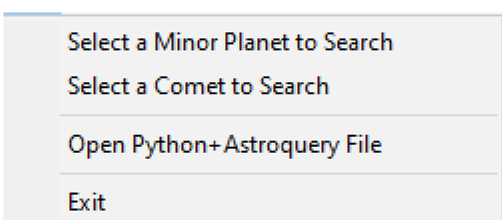
There are many other features in this program for you to discover. Be careful to set the dates on the main screen to the dates obtained in the Sun, Moon, and Planet dialog boxes.

2: The Minor Planet and Comet Search Utility

The Minor Planet and Comet Search Utility is also a free program that can be used to search the Minor Planet Center for data regarding asteroids or comets. The types of data include orbital elements, observations and ephemeris information.

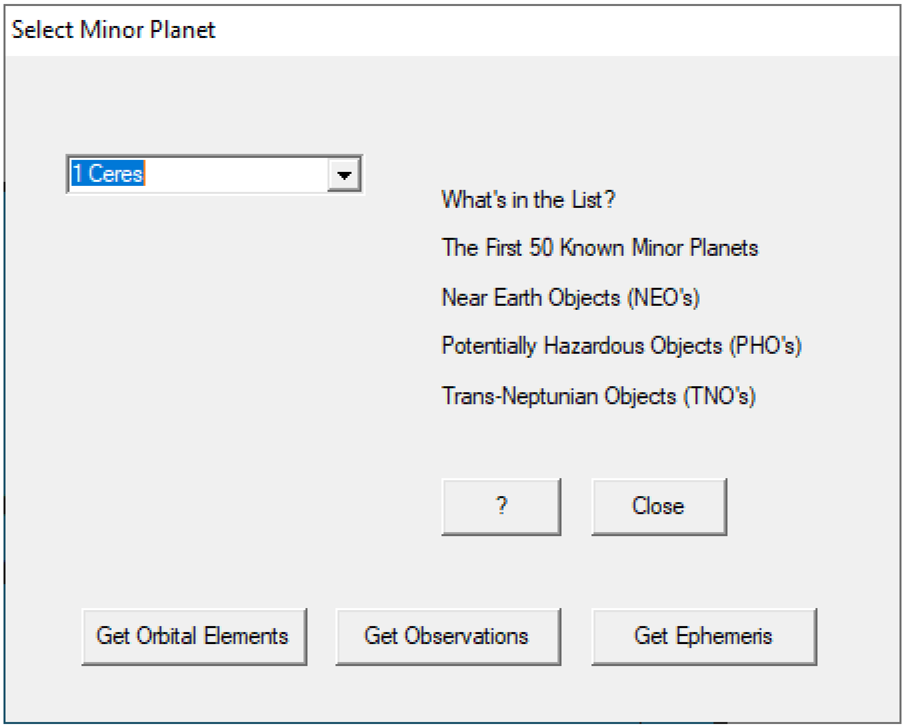


File

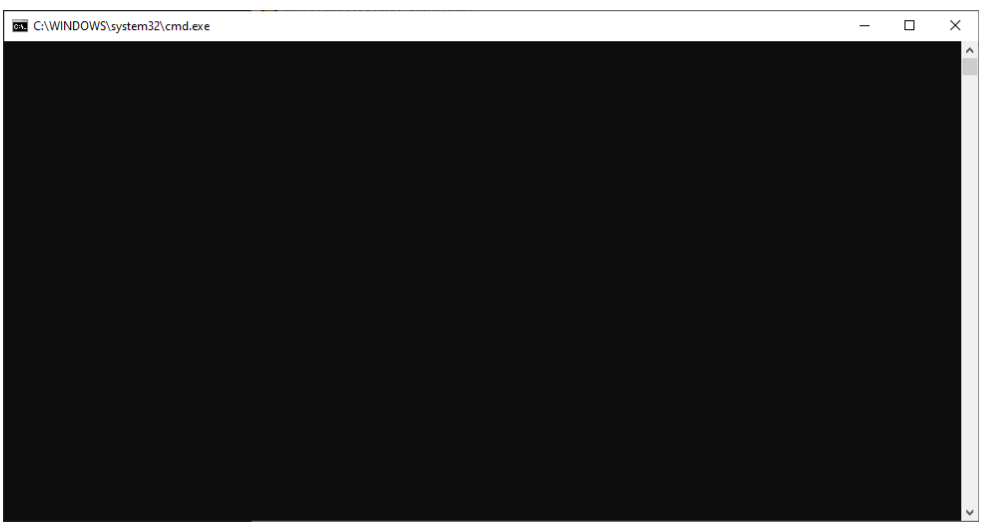


The **File** menu item allows you to search for a minor planet (asteroid) or comet.
The item **Open Python + Astroquery File** allows you to essentially create your own queries for the Minor Planet Center (MPC).

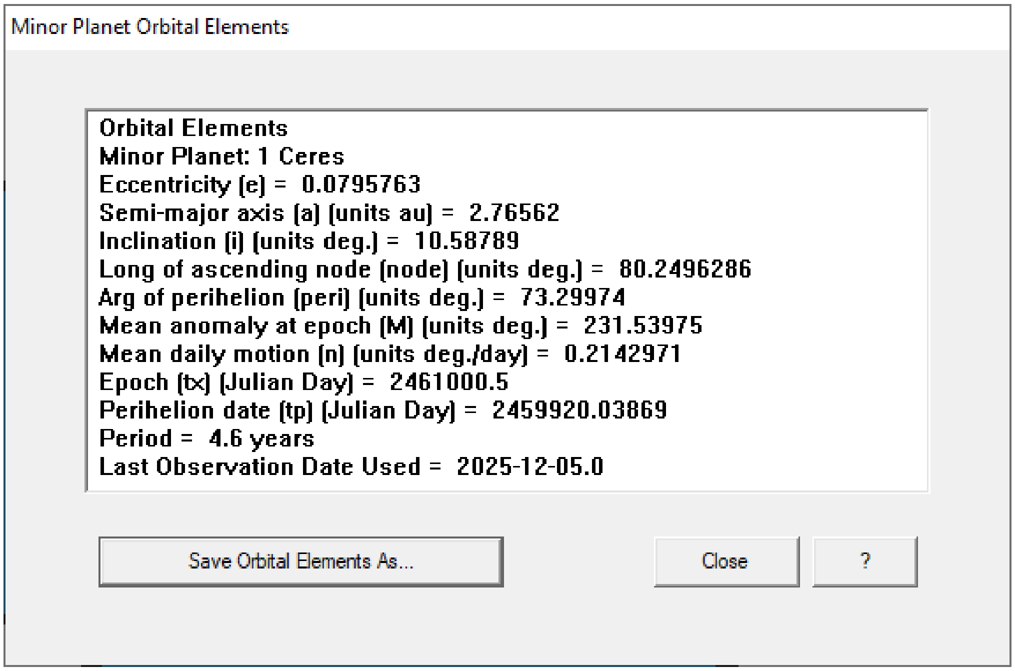
Select a Minor Planet to Search allows you to choose an asteroid from a drop-down list and then get its orbital elements, list of observations or an ephemeris. It is a 'limited' list of asteroids, so that's why there is the possibility of creating your own query for the MPC.



Click the button to **Get Orbital Elements.** The program will use a python script to contact the Minor Planet Center and get the requested data. The screen will be black for several seconds during the time it is contacting the MPC.

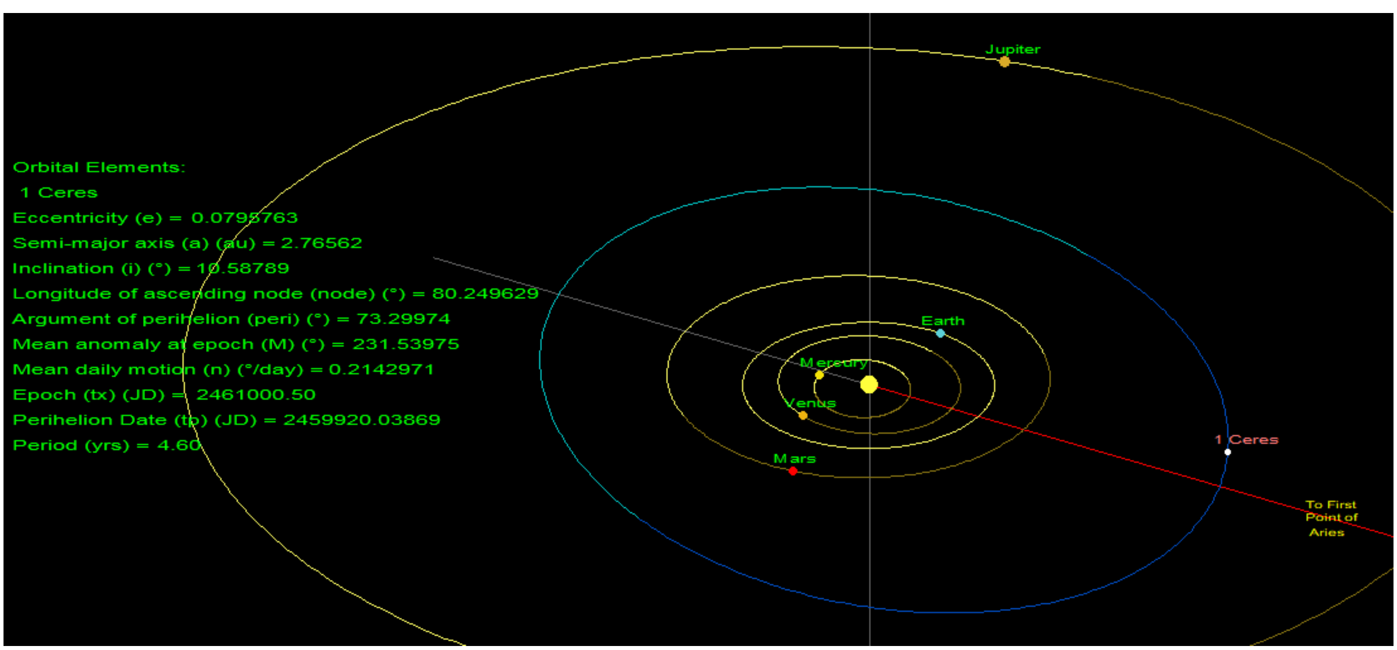


Once it has the data, it will return with a text box showing the orbital elements.



Only the first six items in the orbital elements are actually necessary (plus the **Epoch**). The remaining items are just for general information. These orbital elements are discussed in a later section.

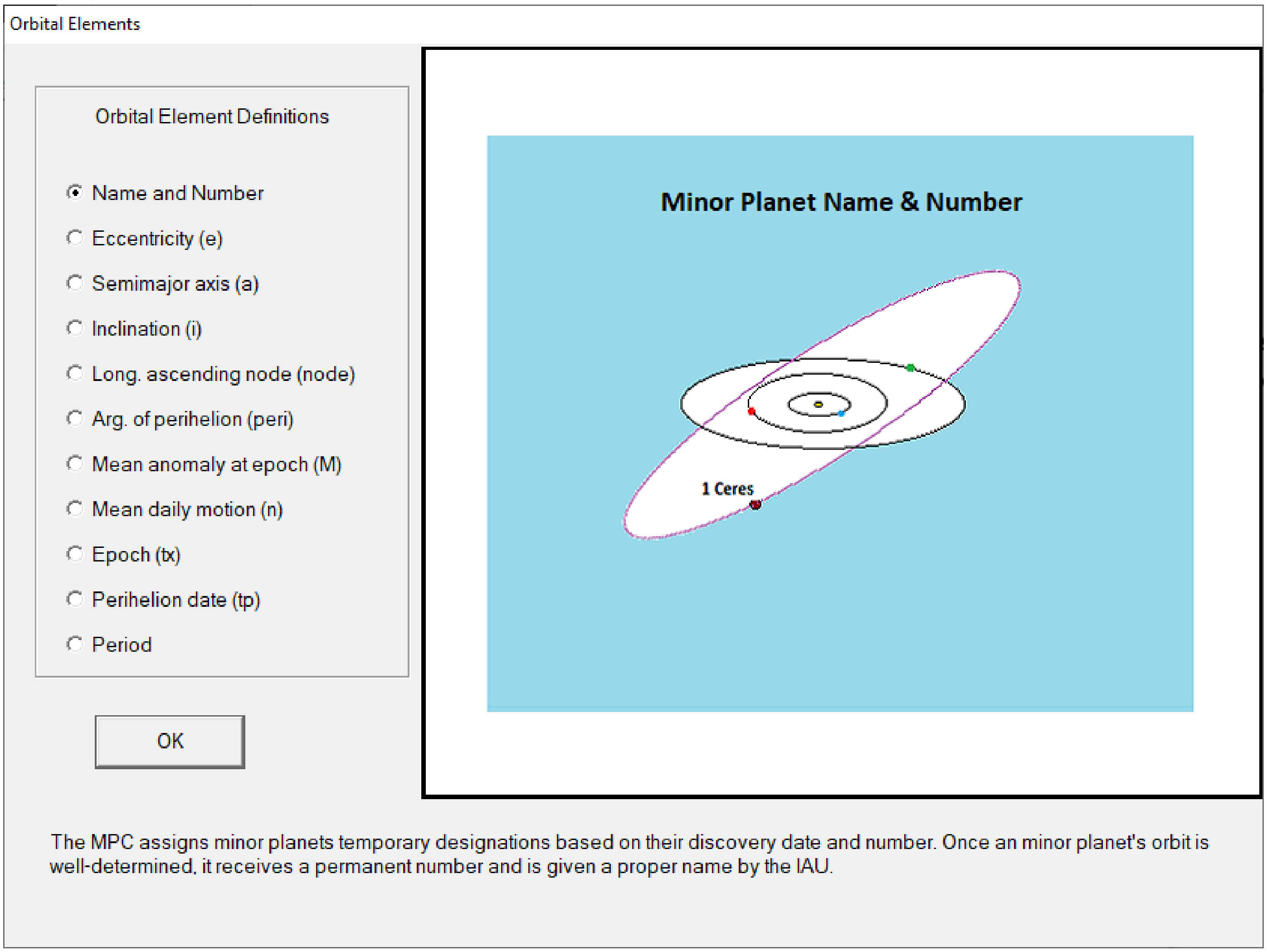
You may and probably should **Save the Orbital Elements.** Put the saved .dat file somewhere on the Desktop or in the same folder as **Sun Moon Planets Simulation**. Next, open the program **Sun Moon Planets Simulation**, go to **File** and **Open Minor Planet Orbital Elements File.** Open the file that you just saved and you should see the orbit of that minor planet (Ceres) on the screen along with its orbital elements.



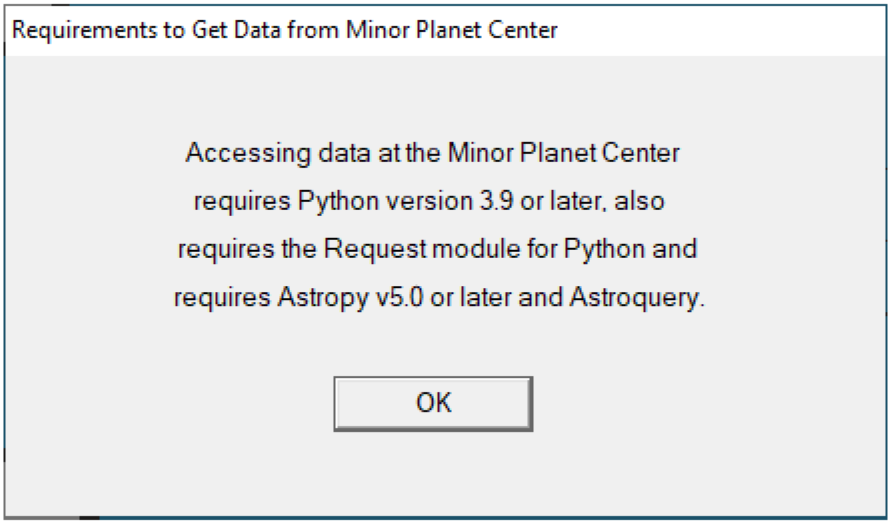
Back to the Search Utility - **Select a Comet to Search** is essentially the same as for Minor Planets above but with a lesser choice of comets.

Help

The **Help** menu item contains a few items that you might find interesting. One in particular is the explanation of orbital elements.



There is also one <u>very important</u> item in the **Help** menu. That is **Requirements to Get Data from Minor Planet Center**. Make sure you read this and install the necessary python applications.



3. Measures of Time – Julian Day Number

The Julian Day Number (JD) is a continuous, sequential count of days and fractions since noon Universal Time (UT) on January 1, 4713 BC.

The Julian Day Number is used mainly in astronomy for easier timekeeping rather than calendar dates, which involve different number of days in a month and leap years.

Calculations between two different times, intervals or events using Julian Days is done by simple subtraction.

A Julian Day starts at noon, not midnight, and uses floating-point numbers for fractions of a day.

For example:

Dec. 16, 2025, at 12 hours (noon), 0 min, 0 sec is equal to 2461026.0 JD

Dec. 17, 2025, at 0 hours (midnight), 0 min, 0 sec is equal to 2461026.5 JD

Dec. 17, 2025, at 12 hours (noon), 0 min, 0 sec is equal to 2461027.0JD

Dec. 18, 2025, at 0 hours (midnight), 0 min, 0 sec is equal to 2461027.5 JD

Why do Julian Days start in 4713 BC?

The Julian Period is a chronological system of timekeeping, invented by Joseph Scaliger in 1582.

It is based on the alignment of three cycles

- A 28-year solar cycle (the period after which calendar dates and weekdays repeat in the Julian calendar).
- A 19-year lunar cycle or Metonic cycle (after which moon phases approximately repeat on the same dates).
- A 15-year indiction cycle (a Roman tax cycle).

January 1, 4713 BC (Julian calendar) at noon was simultaneously the beginning of all three cycles.

The product of the three cycles 19, 28, and 15 is 7,980. It will take 7980 years until all three cycles repeat.

This means AD 2025 is year 6738 of the Julian Period (4713 + 2025 = 6738).

The Julian Day Number is obtained by counting the number of days from January 1, 4713 BC, at Noon, GMT.

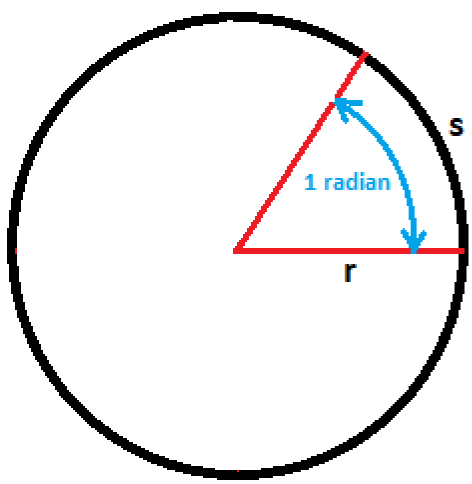
We will be discussing Julian Day Numbers in the following sections.

4. Degrees and Radians

Degrees are generally easier to understand than radians. Degrees are a measure of an angle, 90°, 180°, 270°, 360°. But degrees do not tell you anything about length of an arc on a circle, or any length.
Whenever we need to solve any equation that involves more than just an angle, we use radians.

For example: the circumference of a circle is 2 π r where 2 π = 6.28...
and r is the radius of the circle. The circumference is a length. To say a circle is 360° tells us nothing about the circumference.

1 radian is when the arclength **s** is equal to the radius **r** as shown below.



As you can see, radians depend on the length of **r** and **s**.

If you want to compare degrees and radians, then use the following equation:

$$\frac{180°}{3.14} = \frac{180°}{\pi} = 57.32$$

Or, in other words, 1 radian is equivalent to 57.32°

So, 3.14 radians = π radians = 180°

and 2 π radians = 360°

Converting degrees to radians: Degrees x (π /180) = Radians

Converting radians to degrees: Radians x (180/ π) = Degrees

A degree may also have sub-parts, stated as minutes and seconds.

A degree can be stated in decimal form, such as 30.5°, or as Degrees, Minutes, Seconds (DMS): The DMS form is a sexagesimal (base-60) system where a degree is divided into smaller units. There are 60 seconds in a minute and 60 minutes in a degree.

Comparing the decimal form to the DMS form: 30.5° = 30° 30′ 0″.

In astronomy, when measuring **Declination**, the units are degrees (°), arcminutes (′), and arcseconds (″).

A variation of the DMS system in astronomy is used when measuring **Right Ascension**, it is common to use hours, minutes and seconds where 60 seconds equals a minute and 60 minutes equals an hour, same as on a clock. In this case, an hour would be equal to 15°.

We will see more of this in the next section.

5: What are Orbital Elements?

For any object orbiting around the Sun, its real orbit changes over time due to gravitational perturbations and the effects of relativity. By ignoring these physical effects and treating the orbit in a simpler sense we can specify a set of orbital elements used to describe the position of an object such as a planet, asteroid, or comet as it orbits around the Sun. In this arrangement there are only the two bodies, the object and the Sun; there are no other external physical influences on the motion of the object.

These types of orbits considered in this program are described as classical two-body systems and are based on the six Keplerian elements:

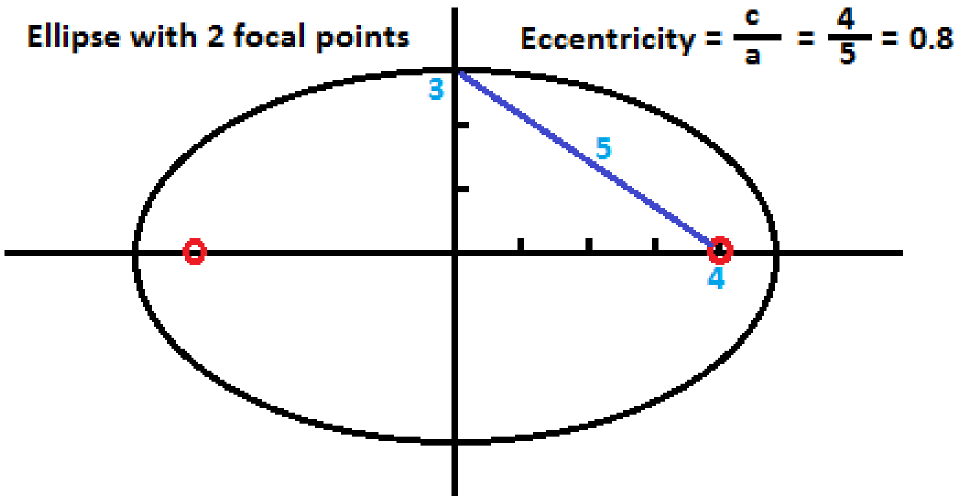
- **Eccentricity (e)** - the shape of an ellipse as compared to a circle.
  For a circle, e = 0; e increases as the circle becomes more elliptical.
  If e = 1 the orbital path is parabolic and if e > 1 it is a hyperbola (these geometric shapes are not considered in this program).
- **Semi-major axis (a)** - add the perihelion and aphelion distances and divide by 2. This is the average distance between the Sun and the object.
- **Inclination (i)** - this is the tilt of the ellipse with respect to the ecliptic.
  It is measured at the ascending node.
- **Longitude of ascending node (Ω)** - this is the point where the orbiting object passes upward through the ecliptic.
- **Argument of perihelion (ω)** - the perihelion is the closest point that the object comes to the Sun. The argument of perihelion is the angle from the ascending node to the point of perihelion.

- **Mean anomaly (M)** - this is the angle from the perihelion to a point along the path of the orbit. The angle is based on the mean average speed of the object and the amount of time it has been travelling on that orbital path since it left the point of perihelion. The mean anomaly is only a guess at where the object is because the speed of the object is not constant - it varies as the object orbits around the Sun.
  The mean anomaly is usually specified at a particular time and date (epoch). The mean anomaly at epoch states the position of the orbiting object along the ellipse at a specific time.

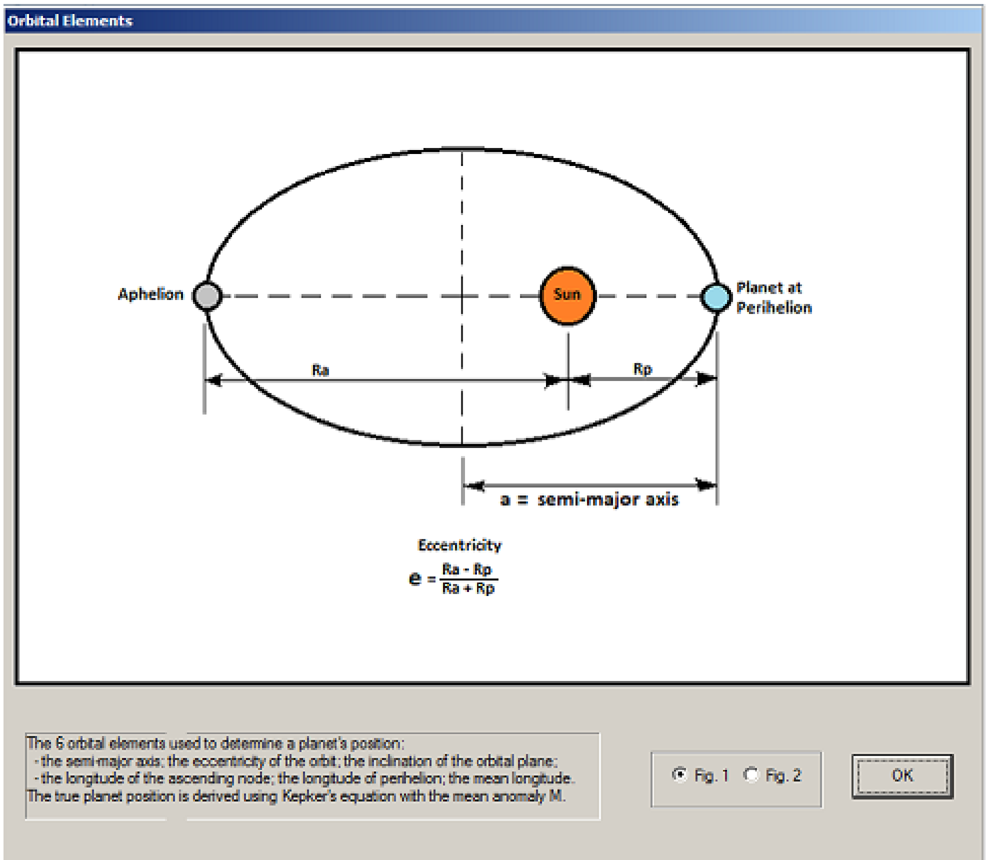
- **True anomaly (ν)** is not usually specified as one of the six orbital elements. The true anomaly is the actual position of the object along the orbital path. The true anomaly is solved with **Kepler's Equation (M = E - e sin E)** where **E** is called the **Eccentric anomaly**.

A graphical way of viewing orbital elements.

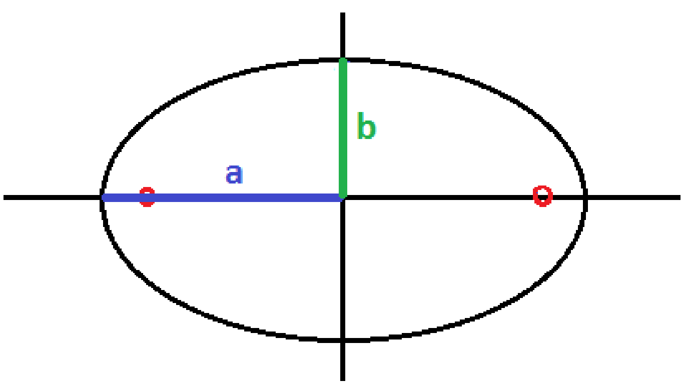
Eccentricity (e) of an Ellipse



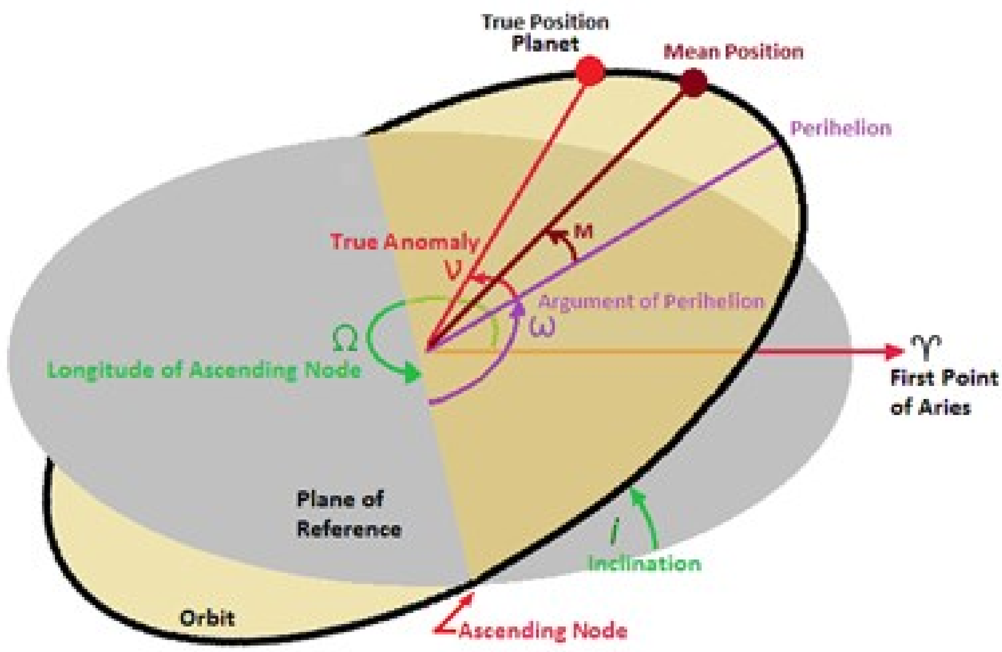
In the **Sun Moon Planets Simulation**, go to the **Help** menu and **Illustrations**
Click the third button down (**Perihelion**)



Note the different way of calculating eccentricity than the method shown above.



Semi-Major Axis (a) and Semi-Minor Axis (b)



Inclination (i), Longitude of Ascending Node (Ω), Argument of Perihelion (ω), Mean Anomaly (M), and True Anomaly (ν)

Notice the red line starting at the center and pointing to the right towards the **First point of Aries** (**Vernal Equinox).** This is the point at which the Sun crosses the Celestial Equator as it moves from the south to the north along the ecliptic.

Starting at the **First Point of Aries** observe the green arc going around to the **Ascending Node**. This is **O**mega (**Ω**), the **Longitude of the Ascending Node.** Sometimes, instead of **O**mega or **Ω** the letter **N** is used. Think **N** for **Node**. Occasionally, the word **node** is used.

Next, observe the arc from the **Ascending Node** to the **Perihelion**. This is **o**mega (**ω**). It is called the **Argument of Perihelion**. Occasionally, the word **peri** is used.

Next, observe the arc from the **Perihelion** to the **Mean Position**.
This angle **M** is called the **Mean Anomaly**.

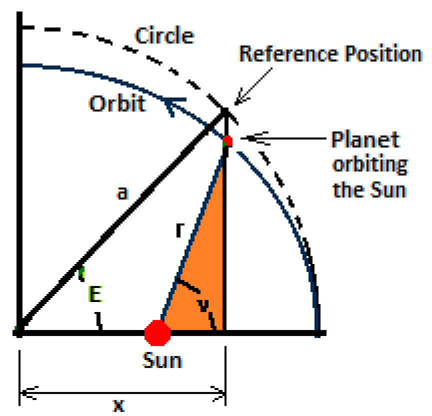
The **Date of Elements** is often called the **Epoch**. The **Mean Anomaly** at **Epoch** states the position of the orbiting object along the ellipse at a specific time.

Note that the **Mean Position** is close to the **True Planet Position**.

The angle of the true planet position is called the **True Anomaly. T**o get the **True Anomaly**, we first need to determine the **Eccentric Anomaly (E).**

The **Eccentric Anomaly (E)** is solved with **Kepler's Equation (M = E - e sin E)** where **E** is the **Eccentric Anomaly**, **M** is the **Mean Anomaly** and **e** is **Eccentricity**.
The **Eccentric Anomaly (E)** is solved using iterative numerical methods since it is a transcendental equation with no direct algebraic solution for the eccentric anomaly (E). We will discuss the numerical methods later.

The relationship between the **Eccentric Anomaly** and the **True Anomaly** is shown below. Once you have the **Eccentric Anomaly**, you can determine the **True Anomaly**.



v is called the True Anomaly
E is called the Eccentric Anomaly

The angle of the Reference Position is called the Eccentric Anomaly (E) and angle of the Planet position is the True Anomaly (ν).

They are related by:

$x = a \cos(E)$, also

$x = r \cos(\nu) + e\, a$

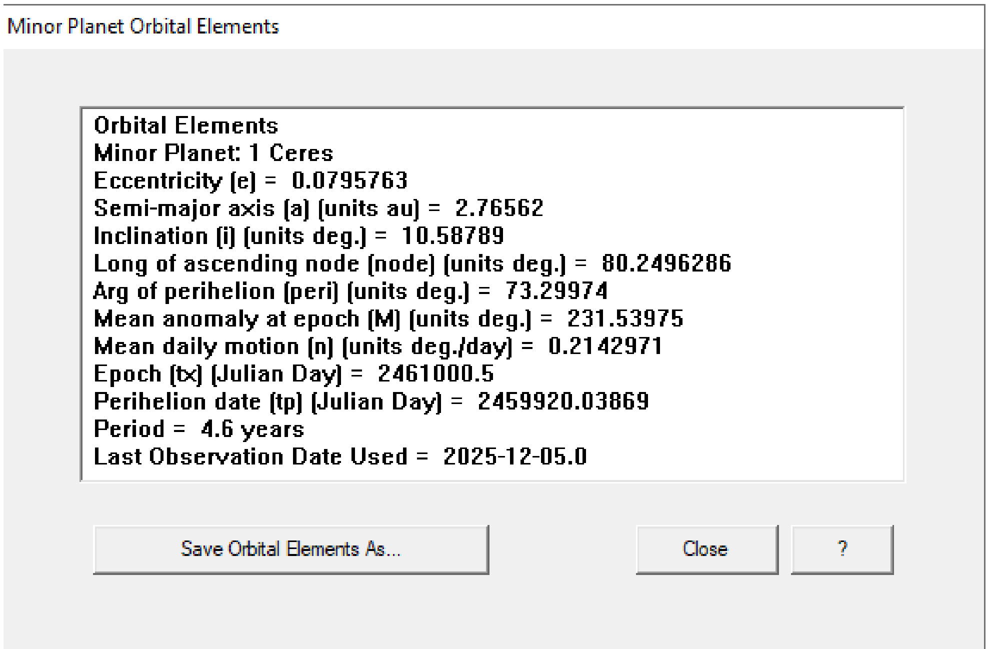
a is the semi-major axis and e is eccentricity

r is the distance from the Sun to the planet

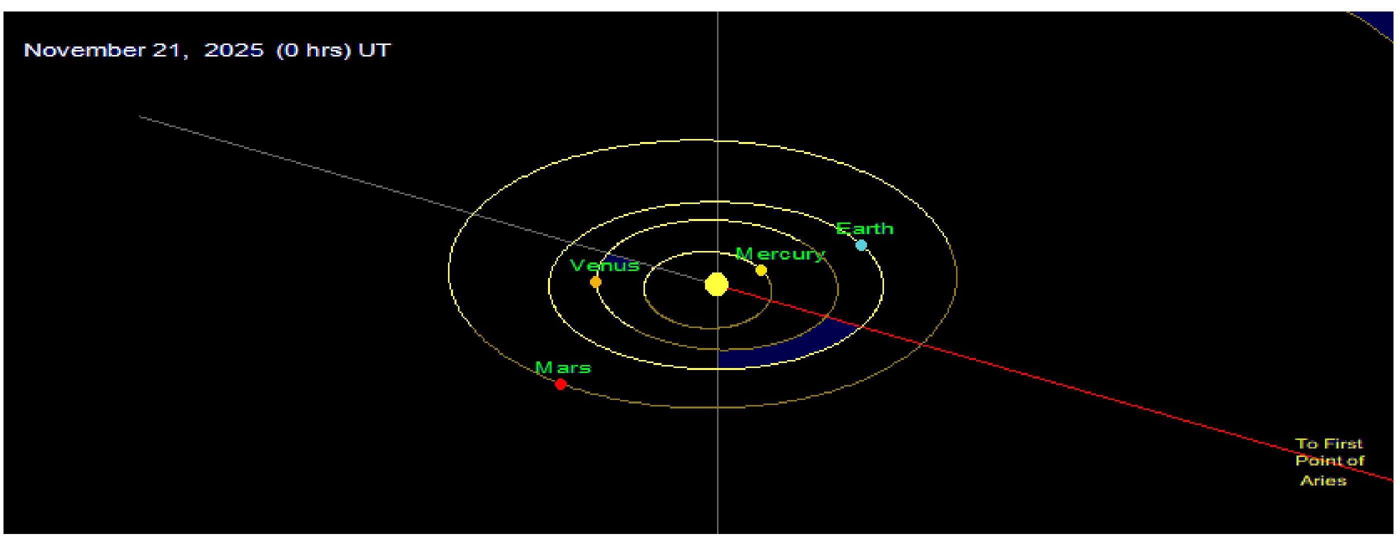
Now lets apply these ideas to an orbiting minor planet

Consider the orbit of minor planet Ceres

Open the program, ***Minor Planet and Comet Search Utility***, click **File** and **Select a Minor Planet to Search.** In the dialog box, combo-box you see **Ceres.** Click the button to **Get Orbital Elements.** It may take a minute to contact the ***Minor Planet Center***, but you should get a text box listing the orbital elements of **Ceres** as shown below. Note, the orbital elements for asteroids and comets changes frequently due mainly to gravitational effects from the major planets.

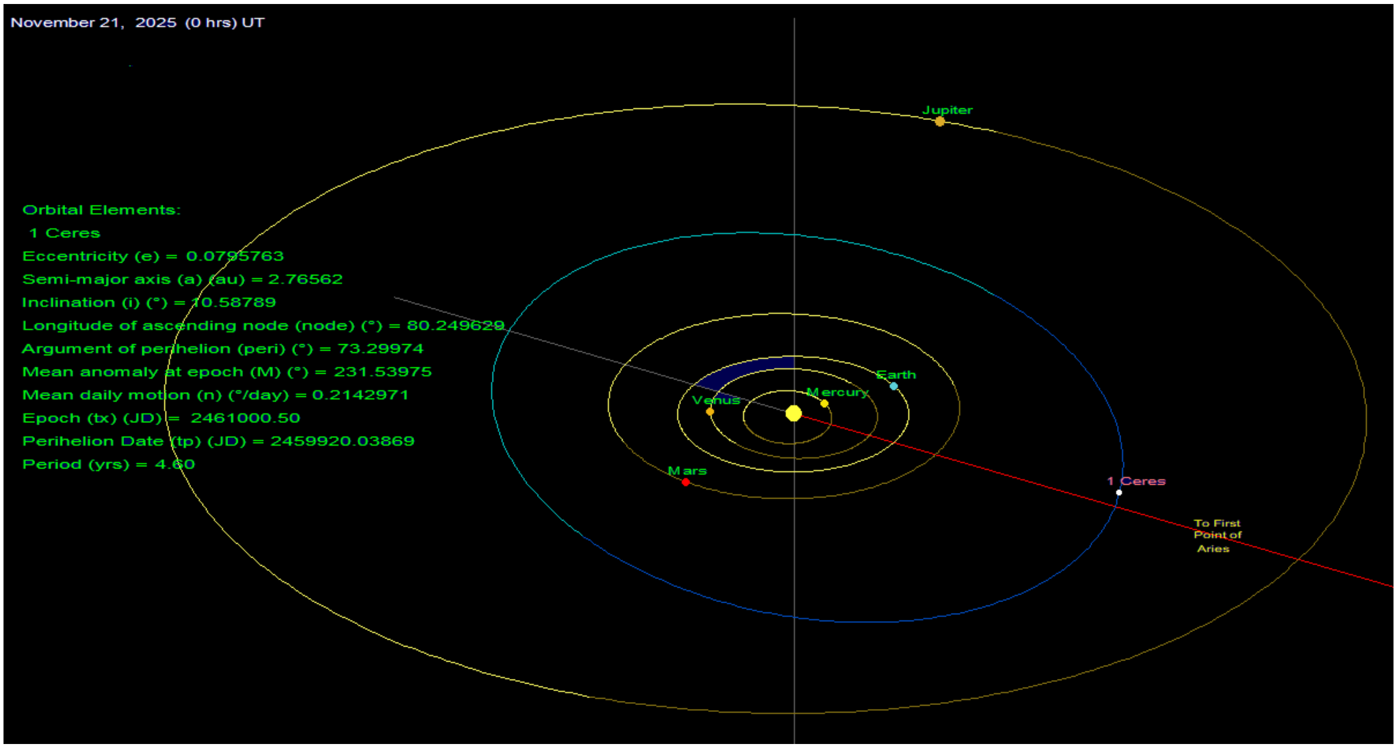


Save these orbital elements to a .dat file and then open up **Sun Moon Planet Simulation**. The **Date of Elements (Epoch)** shown in the listing above is 2461000.5 JD, that is November 21, 2025. Change the date in the **Sun Moon Planets Simulation** (**Tools || Select a Date**) to Nov. 21, 2025.



Use the slider on the right side of the window and move it down so that the **Tilt** angle (shown at bottom of window) is about 30°. This will give a proper perspective for the orbit of **Ceres**.

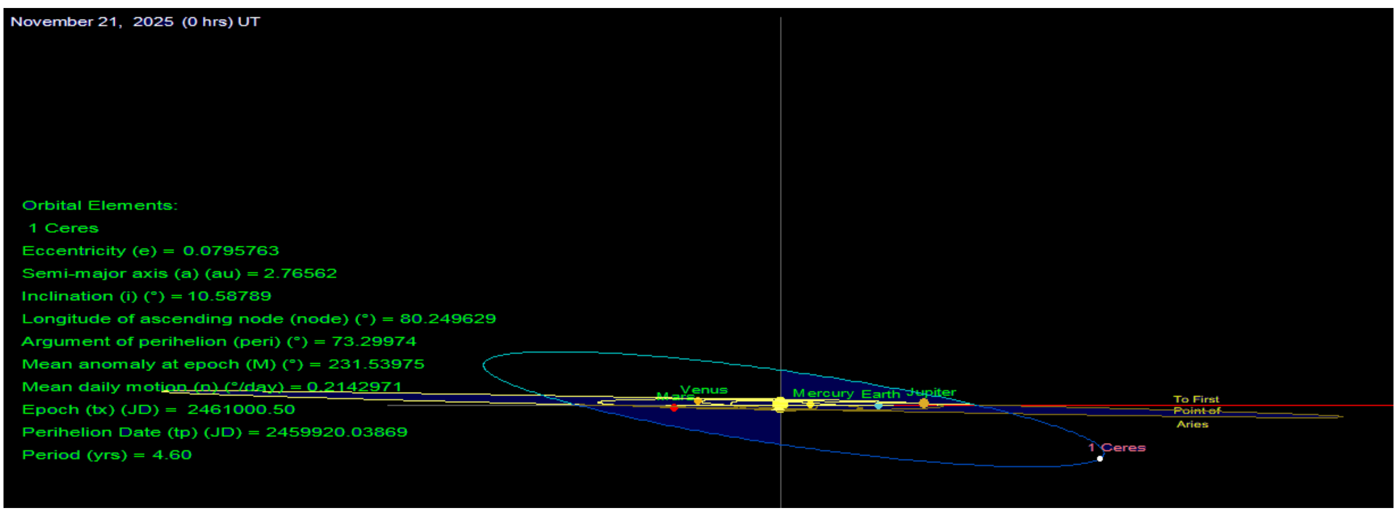
Next, click **File** and **Open Minor Planet Orbital Elements** file. Select the orbital elements .dat file that you previously saved and open it. You should see:



Note **Ceres** on the right-side of the image above and its orbital path in blue. Also, note the orbital elements are listed in green on the left-side.

For now, four of the six orbital elements will be considered. They are **Inclination** (approx. 10°), **Longitude of Ascending Node** (approx. 80°), **Argument of Perihelion** (approx. 73°) and the **Mean Anomaly** (approx. 231°).

We will look carefully at each of these elements, so to begin, use the vertical slider on the right-side of the display and set it to about the middle position. Note along the bottom of the display the **Tilt** angle is 0°. You should see the image below.

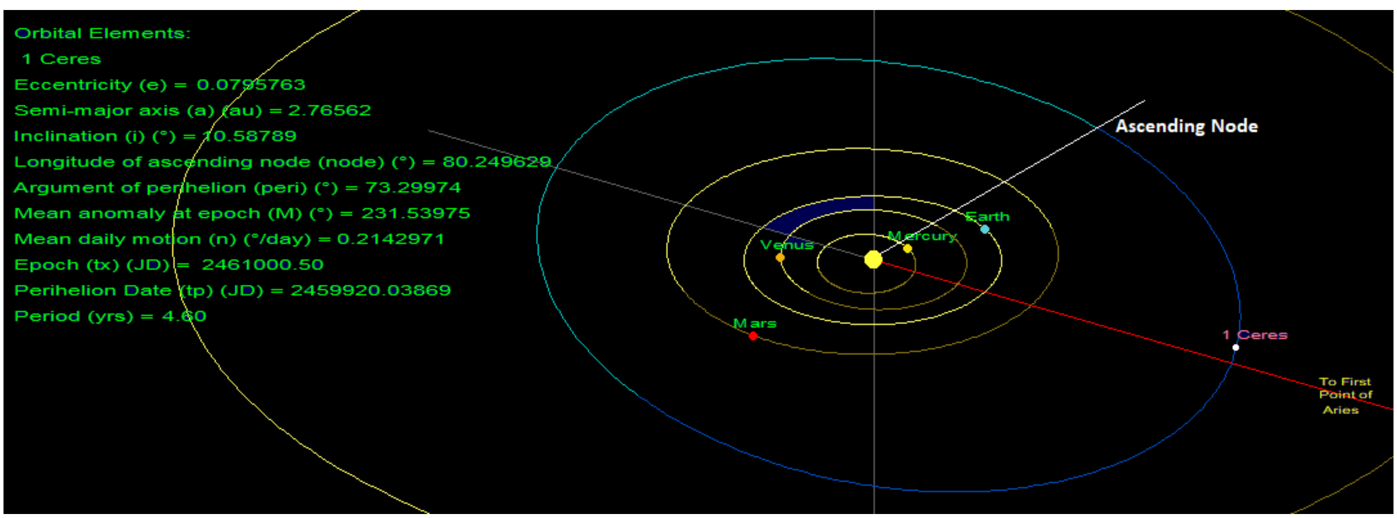


Note the orbit of **Ceres** is shown in blue and that the orbit is inclined at around 10°. The orbital elements (green text) on the left shows it at 10.58789°.

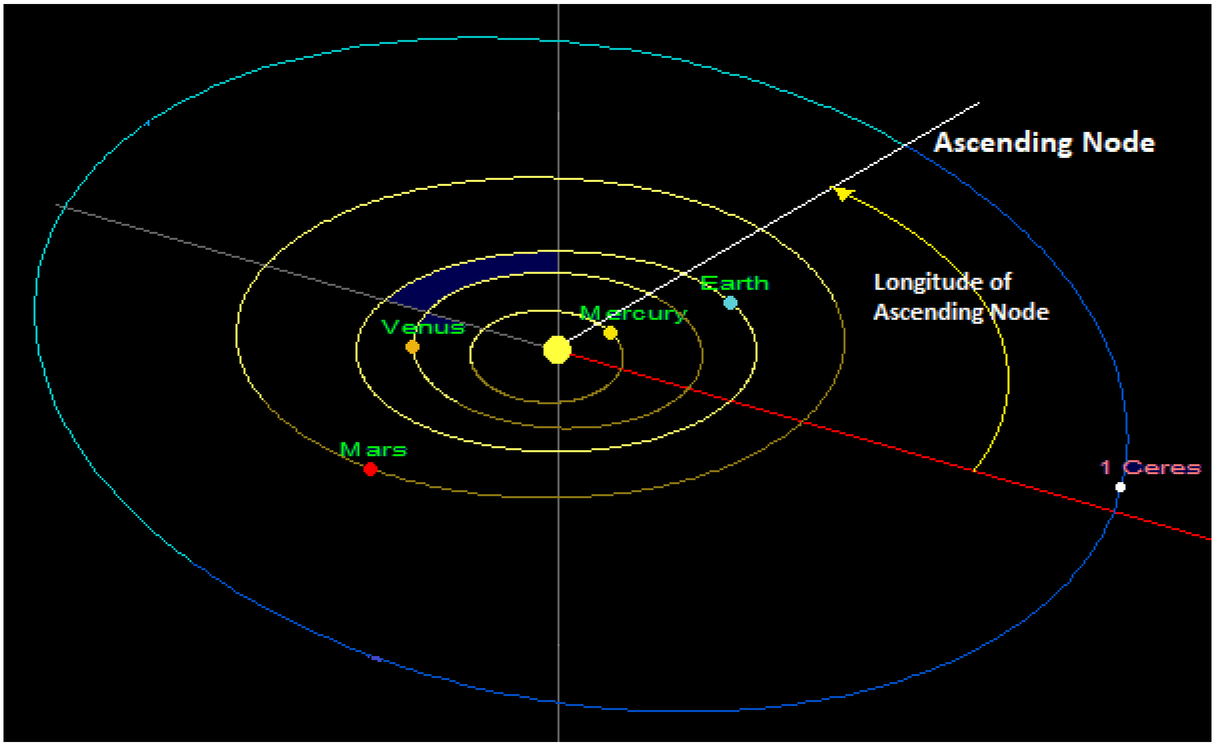
The bright part of the orbital path of **Ceres** as shown on the screen indicates where the orbit is above the ecliptic plane. The darker part of the orbital path indicates where the orbital path is below the ecliptic plane.

Return the vertical slider to a position near the bottom of the display such that the **Tilt** angle is around 30°. This provides a reasonable perspective of where **Ceres** is compared to the other planets.

Now, imagine drawing a line from the **Sun** to the **Ascending Node** (towards the upper right-side of the image where the darker orbital path of **Ceres** changes to the brighter orbital path (the opposite node is **descending** since objects orbit around the **Sun** counter-clockwise as viewed from the top).



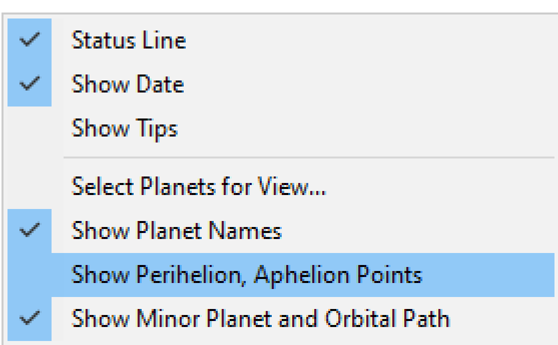
Next imagine drawing an arc starting at the red line (**First Point of Aries**) counter-clockwise around to the line from the **Sun**, as shown below.



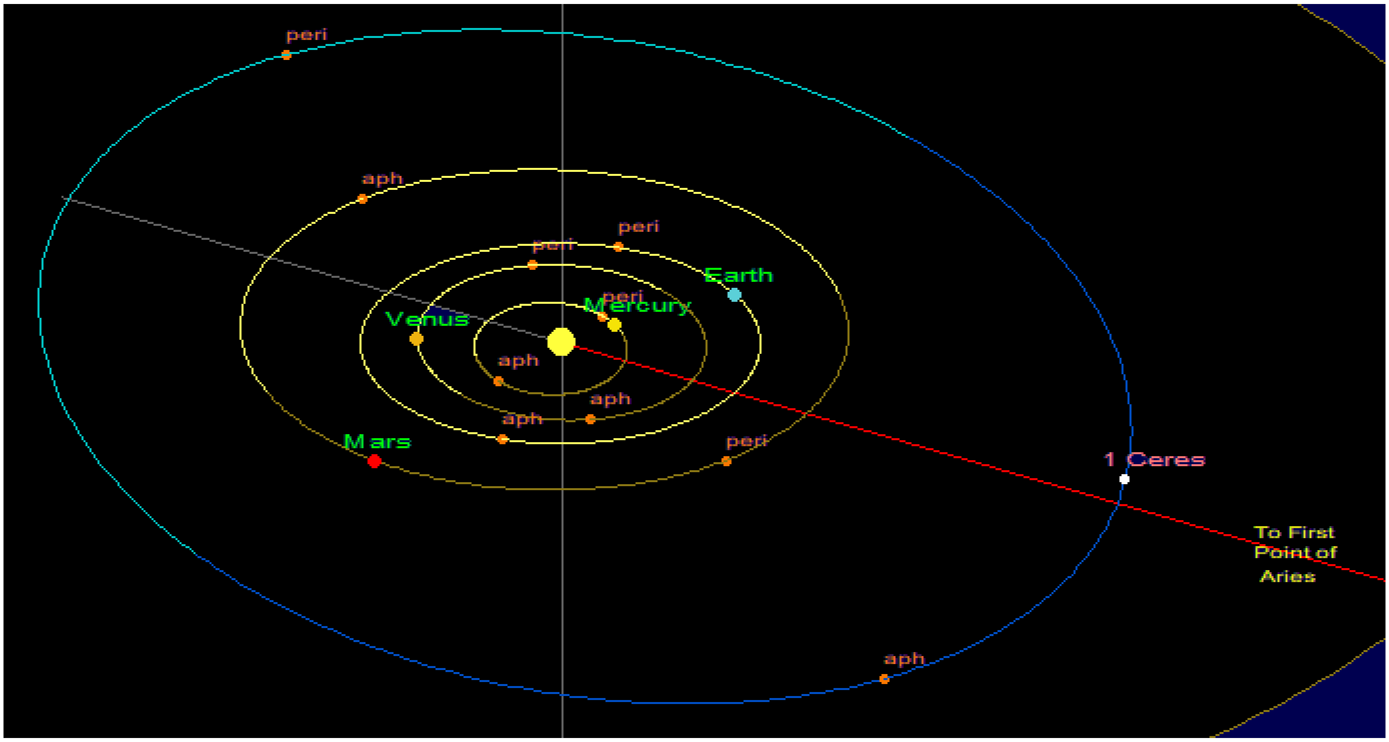
The angle of that arc is called the **Longitude of Ascending Node** and in this case for asteroid **Ceres** you can see in the orbital elements shown previously, that the **Longitude of Ascending node (node)** is approximately 80°. This appears to be visually correct since a quarter circle would is 90°.

It is important to understand that this angle, the **Longitude of Ascending Node** is in the same plane as **Earth** and **Sun** – the **Ecliptic** plane.

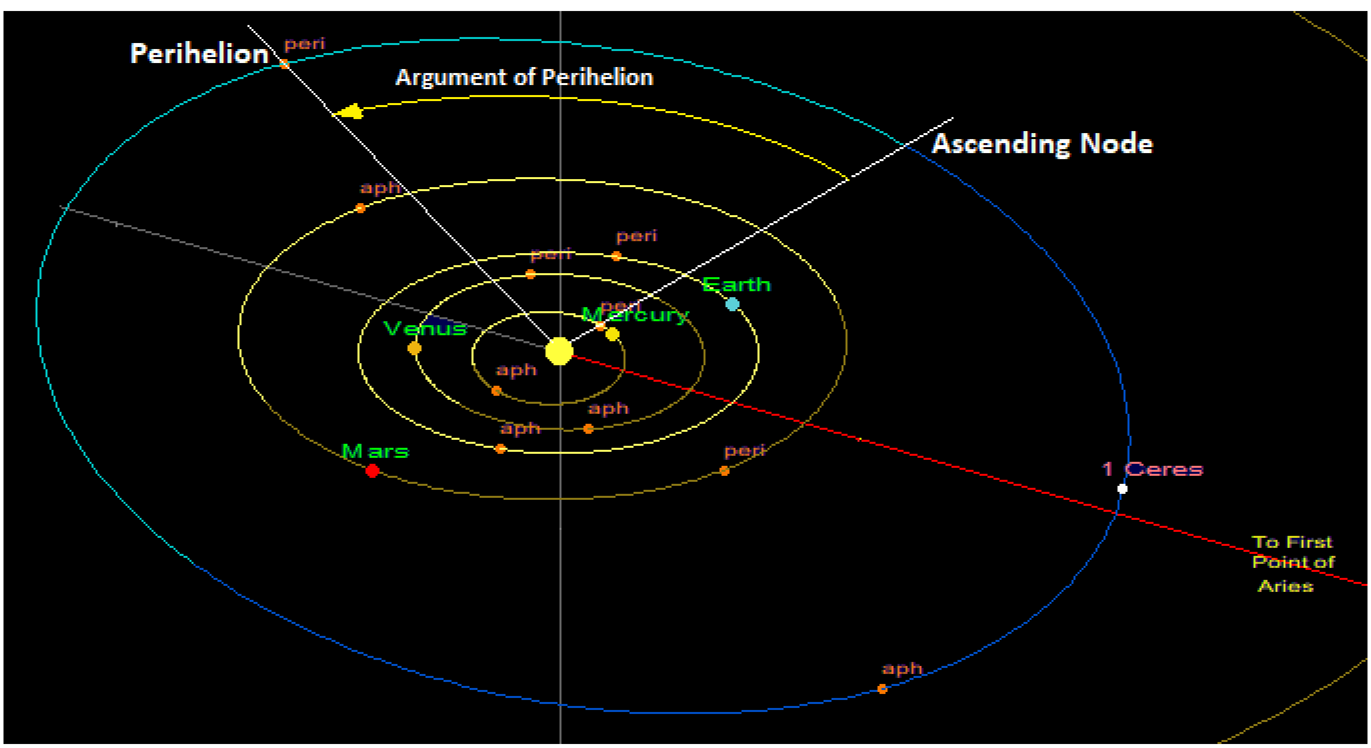
Next, we need to locate the **Perihelion** of the orbit of **Ceres**. This is the point where **Ceres** is closest to the **Sun**. The **Perihelion** of the orbit can be found by going back to the **View** menu and clicking **Show Perihelion, Aphelion Points**.



The screen will then show the following.



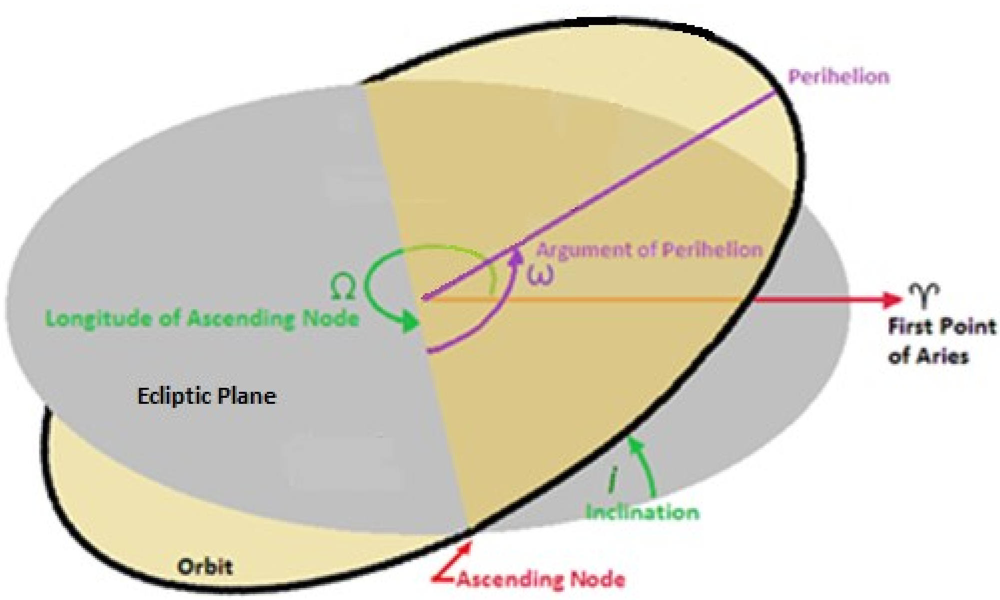
Note the two points on the orbital path of **Ceres** marked **'peri'** and **'aph'**.
Now, imagine drawing a line from the **Sun** to the point of **Perihelion** and then drawing an arc from the **Ascending Node** to that line. That arc or angle is called the **Argument of Perihelion** (**peri**).



If you look in the orbital elements that were shown above, you will see that the **Argument of Perihelion (peri)** is approximately 73°. This again appears to be visually correct.

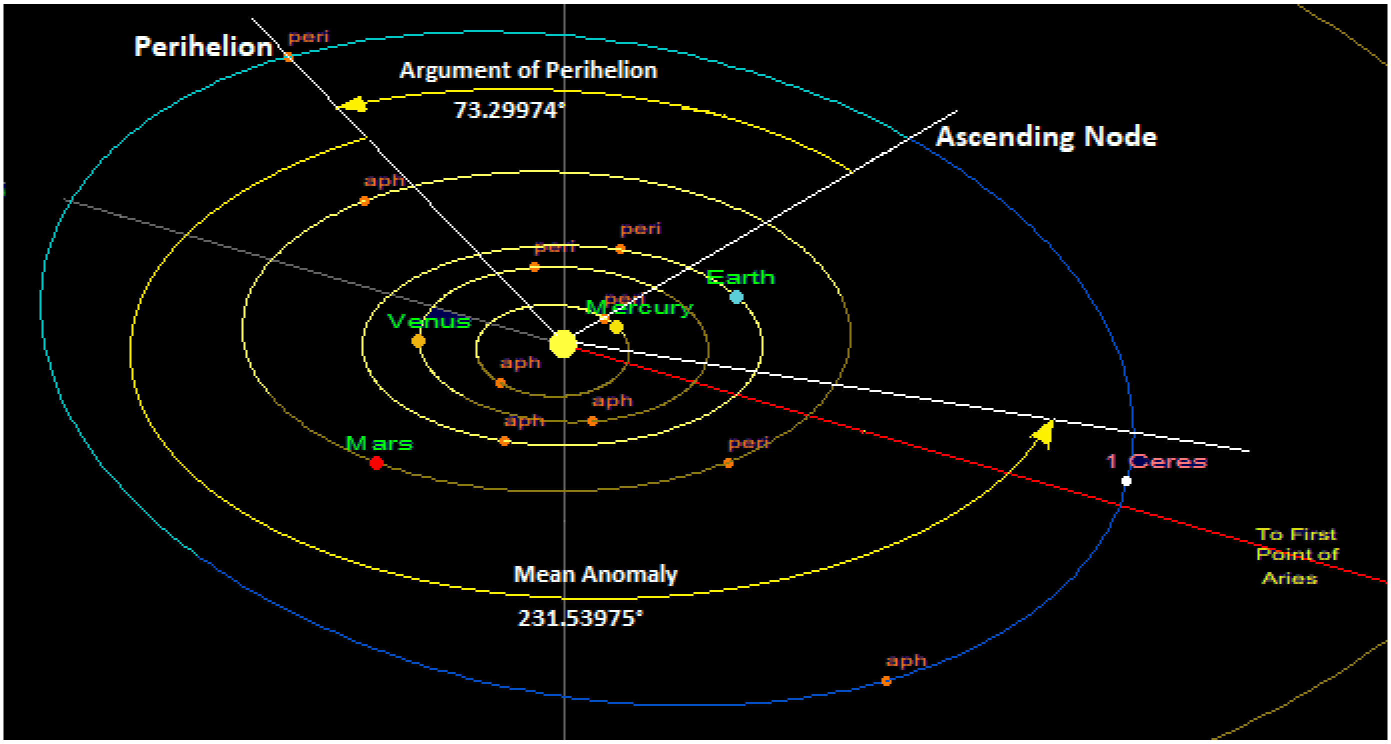
It is important to understand that this angle, the **Argument of Perihelion (peri)** is <u>NOT</u> in the same plane as Earth and Sun – the **Ecliptic** plane. It is <u>NOT</u> in the same plane as the **Longitude of Ascending Node (node)**.

This is important because the **Longitude of Ascending Node** and the **Argument of Perihelion** cannot be added together as a simple sum of angles. Because of the inclination, the two angles exist in different planes (shown below). They can only be added if the orbit of the minor planet is in the same plane as the ecliptic.



The next step is to draw a line from the **Sun** to an imaginary point on the orbital path called the **Mean Anomaly** (**M**). Then draw an arc from the **Perihelion** to that line. That arc or angle is called the **Mean Anomaly** (**M**). Since the **Mean Anomaly** is approximately 231° (at the **Epoch** date 2461000.5 JD) then that arc will continue along the orbit of **Ceres**, through the descending node, into the second half of the orbit and then back into the first half of the orbit of **Ceres**, past the **First Point of Aries**.

Note in the image below, the **Mean Anomaly** point is close to **Ceres**, but not actually at **Ceres**.



We can see that the **Mean Anomaly** is close to the **True Anomaly** (**True Position** of **Ceres**) but not exactly. The **Mean Anomaly** is based on the **mean daily motion (n)** times the number of days since the asteroid passed the **Perihelion**. The problem is that according to **Kepler's Laws**, an orbiting object does not have a constant speed - it goes fast and slow depending on where it is in its orbit. Therefore, we just use an average speed - and that gives us the **Mean Anomaly**. The **True Anomaly** (actual position) is derived by first solving **Kepler's Equation** to get what is called the **Eccentric Anomaly**, and from that we calculate the **True Anomaly**. We will take a look at how to calculate the **Eccentric Anomaly** and the **True Anomaly** next.

Throughout all of this you need to keep in mind that the **Mean Anomaly** above was calculated at the **Epoch** of 2025/11/21 at 0 hours UT (Julian Day 2461000.5). As the days advance **Ceres** continues to move along the orbital path and so these additional days would have to be included in the calculation of the **Mean Anomaly**.

When you combine these orbital elements (**i, Ω , ω , M**) with the remaining other orbital elements (**Semi-Major Axis a, Eccentricity e**) you have a full set of coordinates to describe the heliocentric position of any orbiting object around the **Sun**.

A Bit of Math Regarding the Mean Anomaly

Let **te** represent the **Epoch Date** and **tp** represent the time and date at which the asteroid (or comet) is passing the point of **Perihelion** (the point closest to the **Sun**). From these two dates we calculate the number of days from **Perihelion** to the **Epoch,** ie., number of days = **te - tp** . This is generally calculated using the Julian Day Number for both **te** and **tp**.

Also, let **n** represent the *average speed* that the object is moving (in terms of **degrees per day**). It is also called **Mean Daily Motion**.

Let **Mo** represent the **Mean Anomaly**. This is the angle (in degrees) that the object has traveled from the time of the **Perihelion** (**tp**) to the time of the **Epoch** (**te**). Use the equation: **Mo = n * (te - tp)** to calculate the **Mean Anomaly**.

Mo is the value of **M** at the **Epoch**.
tp is the time of **Perihelion** passage (in Julian Day Number).
te is the number of days from **tp** to the **Epoch Date.**
n is the average speed (degrees per day) or **Mean Daily Motion**.

The **Period** (**P**) of the orbit is the number of days it takes for the object to make a complete orbit around the Sun. The **Period** can be calculated using one of Kepler's Laws: The square of the period **P** of any planet is proportional to the cube of the **Semi-Major Axis (a)** of its orbit. Therefore, solve for **P** given $P^2 = a^3$

When we calculate **P** in terms of Earth's orbit: $P = 365.256898326 * a^{3/2}$ days.
If we know the **Semi-Major Axis (a)** of an object, we can solve for **P**; or, we can just count the number of days for a complete orbit.

Now that we have **P**, we can solve for the **Mean Daily Motion** (**n**). Since an object moves through 360 degrees in one orbit then the **Mean Daily Motion** is
n = 360 / Period (degrees per day).

Therefore, **Mo = (360 / Period) * (te - tp)**

Epoch is the date at which all the orbital elements were last calculated (in Julian Day Number).

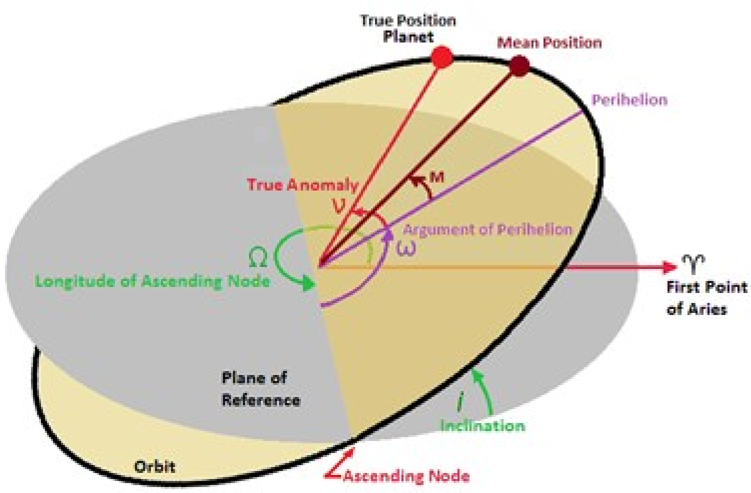
If we want to know the **Mean Anomaly** (**M** not **Mo**) at a later date (the object has moved farther along its orbit) then we use the same type of calculation.

M is the **Mean Anomaly** at any arbitrary date. Instead of **(te - tp)** we use **(t - te)** where **t** is typically the present date (or any arbitrary date).

Then at any arbitrary date, **M = n * (t - te) + Mo** where again **t** is typically the present date. If we combine **(t - te)** and **(te - tp)** then we can skip **Mo** and say:
M = n * (t - tp).

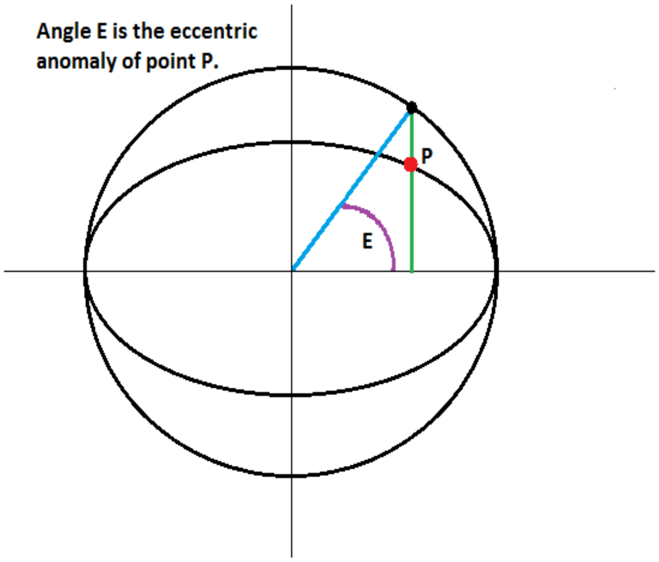
One last detail, if the **Epoch Date** is the same as the **Perihelion Date** (ie. **te = tp**). This means that since **Mo = n * (te - tp)** and **te - tp = 0** then
Mo = 0. In this case you would use **M = n * (t - tp)**.

Now that we understand the **Mean Anomaly**, let's move on to the **Eccentric Anomaly** and the **True Anomaly**, the actual position of the object.



Before we solve for the **True Anomaly (ν)** we need to understand the **Eccentric Anomaly (E)**.

In the figure below, imagine the elliptical path of an orbit with planet **P**, all enclosed by a circle. Draw a line from the center up to the circle such that a triangle is formed with the vertical side passing down through the planet **P**. The angle **E** that is formed is then called the **Eccentric Anomaly**.

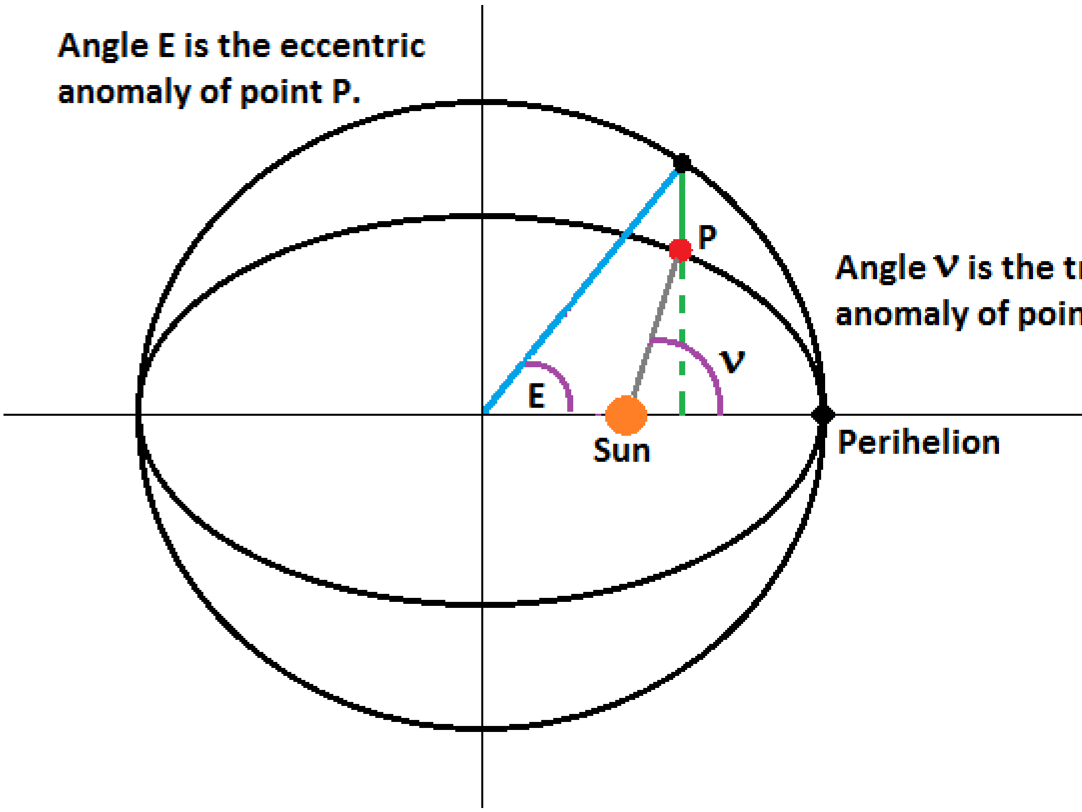


The **Eccentric Anomaly (E)** is related to the **Mean Anomaly (M)** by **Kepler's Equation**:

$$M = E - e * \sin(E)$$ where **M** and **E** are in radians

Once Kepler's Equation is solved for the **Eccentric Anomaly**, the relation between the **Eccentric Anomaly (E)** and the **True Anomaly (ν)** is:

$$\nu = 2 \arctan\left(\sqrt{\frac{1+e}{1-e}} \tan\frac{E}{2}\right)$$ where **e** is **Eccentricity**



Now look at it again to see how it all fits. The **Sun** is at a focal point, not at the center.

If the **Sun** was at the center, the orbit would be a circle and the **Mean Anomaly,** the **Eccentric Anomaly** and the **True Anomaly** would all be the same angle pointing to the same object on the orbital path, but, this is very rarely the case.

For the vast majority of cases where the orbit is not a perfect circle, we need to first solve for the **Mean Anomaly (M)**. Second, use **M** and **Kepler's Equation** to solve for the **Eccentric Anomaly (E)**, and third, use **E** to solve for the **True Anomaly (ν)**.

Kepler's Equation (**M = E - e * sin(E)**) cannot be solved algebraically. It is usually solved by using iteration methods with a computer. This method is called the **Newton-Raphson Method** and involves finding the roots of the equation (see **Appendix**):

$$f(E) = E - e*\sin(E) - M$$

Note: if using a computer program to solve this equation, **E** and **M** are measured in radians, this equation does not work for degrees. To solve for the roots, set **f(E)** = 0

Ok, now let's return to the previous problem of calculating the **True Anomaly** of **Ceres** given the orbital elements as before, and this time our calculations will be more precise instead of just approximations.

Arg of perihelion = 73.29974° = 1.27932 Radians
Mean anomaly at epoch (M) = 231.53975 ° = 4.041131 Radians at **Date of Elements**
Date of Elements (Epoch) is 2461000.5 JD or November 21, 2025.
Eccentricity (e) = 0.0795753

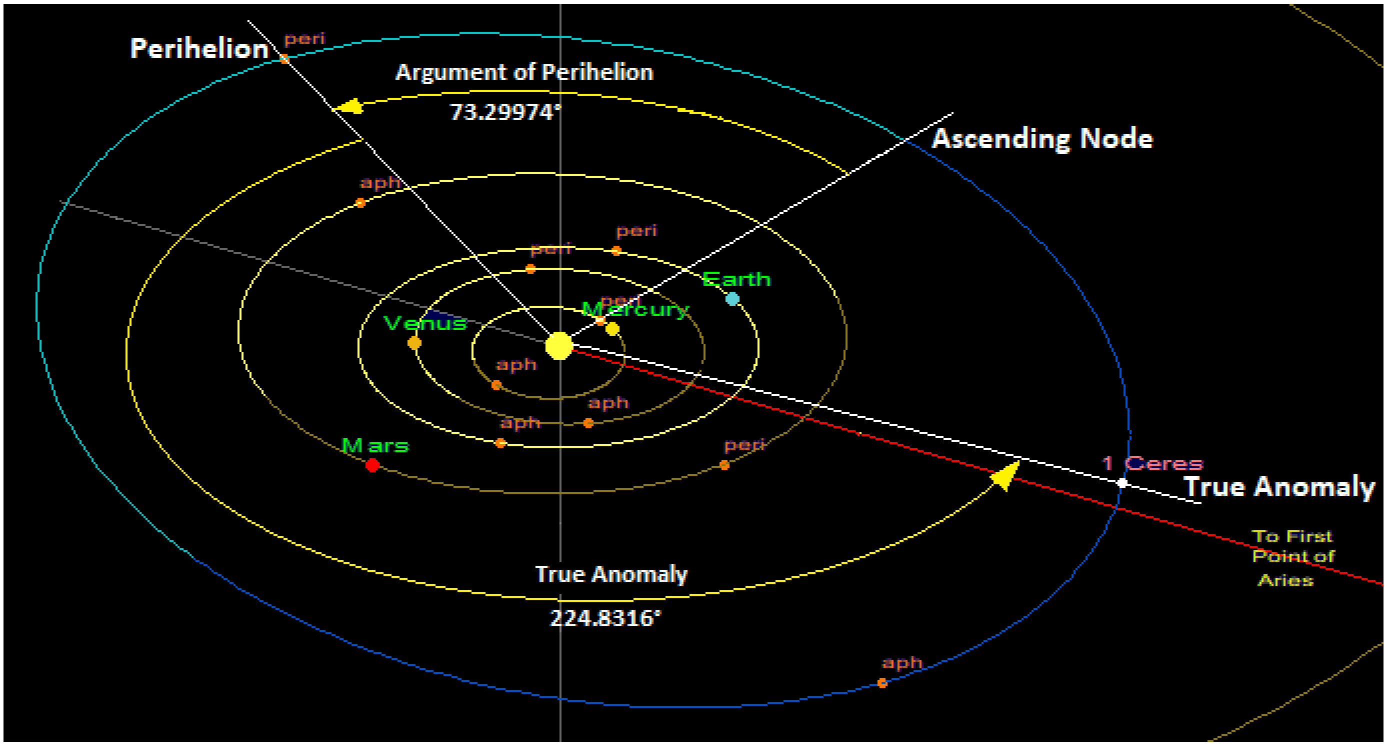
Use **Kepler's Equation** (**M = E - e * sin(E)**) to get **E**
Refer to python code in the Appendix for solving Eccentric Anomaly.
It is found that **E** = 228.14382° = 3.98186 Radians

Now use the **Eccentric Anomaly (E)** to calculate the **True Anomaly**

$$\nu = 2 \arctan\left(\sqrt{\frac{1+e}{1-e}} \tan\frac{E}{2}\right)$$

ν = 2 * arctan(sqrt((1+0.0795753)/(1-0.0795753)) * tan(228.14382/2))
ν = 2 * arctan(sqrt(1.1729099) * tan(114.07191))
ν = 2 * arctan(1.083009 * (-2.23847))
ν = -135.1684°
A negative angle might seem strange. It means going backward (clockwise) from **Perihelion** back to **Ceres**.

To go forward from the **Perihelion** in the direction of the orbit (counter-clockwise), the **True Anomaly (ν)** = 360.0 – 135.1684 = 224.8316°



Take a look at **Sun Moon Planets Simulation** menu item **Orbits || SkyView with Orbiting Planets**. Find **Ceres** (to compare with results shown here, make sure you set the date to Nov. 21, 2025). Also check menu item **Dialogs || Ecliptic View**; again, find **Ceres**.

Now that we have discussed the meaning of all six orbital elements, we next consider the concepts of **Right Ascension (RA)** and **Declination (Dec)**.

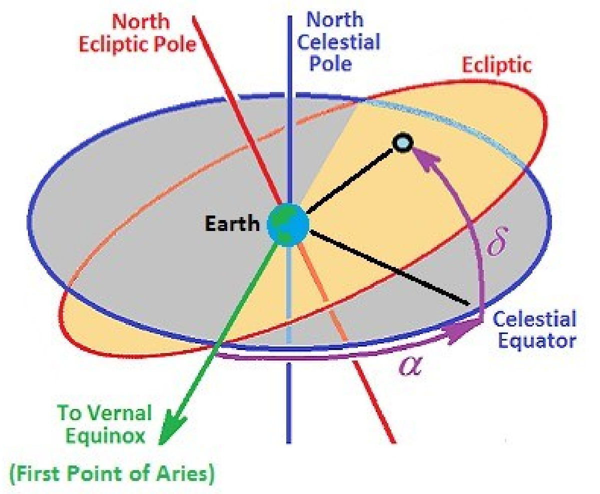
To ty the orbital elements to **RA** and **Dec**, when we check the Minor Planet Center ephemeris for **Ceres** on Nov. 21, 2025, the Right Ascension is 0h 27m 15.5s and the Declination is -10° 12**'** 3**"**

Remember, the **First Point of Aries (Vernal Equinox)** is defined as RA = 0 hours, 0 minutes, 0 seconds (0h 0m 0s) and Dec = 0 degrees, 0 arcminutes, 0 arcseconds (0° 0' 0"). So, since **Ceres** is very close to that, all looks well.

6: Right Ascension and Declination – An Overview

Right Ascension (**RA** or Greek letter **α**) is the angular distance of an object (like a planet or star) that is east of the **First Point of Aries** (**Vernal Equinox** or ♈) and is measured along the **Celestial Equator**. The angle is measured in hours, minutes and seconds (h,m,s).

Declination (**Dec** or Greek letter **δ**) is the angular distance of an object that is north or south of the **Celestial Equator**. The angle is measured in degrees (°), minutes ('), and seconds ("). The **Celestial Equator** is 0° **Dec** and the poles are +90° (north) and -90° (south).



Right Ascension (α) and Declination (δ)

Where is the **First Point of Aries**? It is now in the constellation Pisces. Just click menu item **Dialogs** and select **Ecliptic View**. In the dialog box note the location of RA = 0 hours and Dec = 0°. You should see the constellation Pisces just above that point.

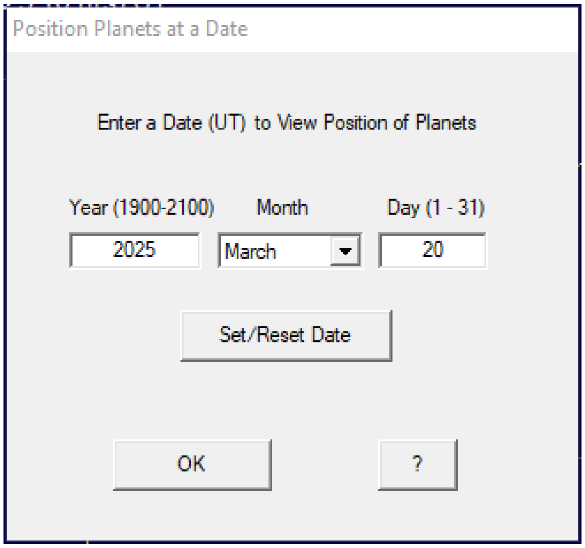
In the figure above there are several items that need to be clearly understood.

- Imagine that the Earth is at the center. The equator of the Earth extends out into space as the **Celestial Equator**. The north and south poles of the Earth also extend out as the **North Celestial Pole** and the **South Celestial Pole** (see blue line above).
- The Earth revolves around the Sun in a plane that is tilted at approximately 23.5° with respect to the **Celestial Equator**. This plane is called the **Ecliptic**. The **Ecliptic** also has a **North Ecliptic Pole** and a **South Ecliptic Pole** (see red line above).
- Note the point where the **Ecliptic** rises through the **Celestial Equator**. Observe the green line from the Earth to that point. That line points in the direction of the **First Point of Aries** (**Vernal Equinox** or ♈). That direction is where the **Right Ascension (RA)** is equal to 0h, 0m, 0s and the **Declination (Dec)** is 0°, 0', 0".
- The **First Point of Aries** is our starting point. Now imagine a planet or star in the sky. The coordinates of that object would be stated in terms of **RA** and **Dec**. To get **RA**, move counter-clockwise from the **Vernal Equinox** along the **Celestial Equator**. That angle is shown in the figure above as **α.** Next, move up (northward) from the **Celestial Equator** to the object. That angle is shown in figure above as **δ**.

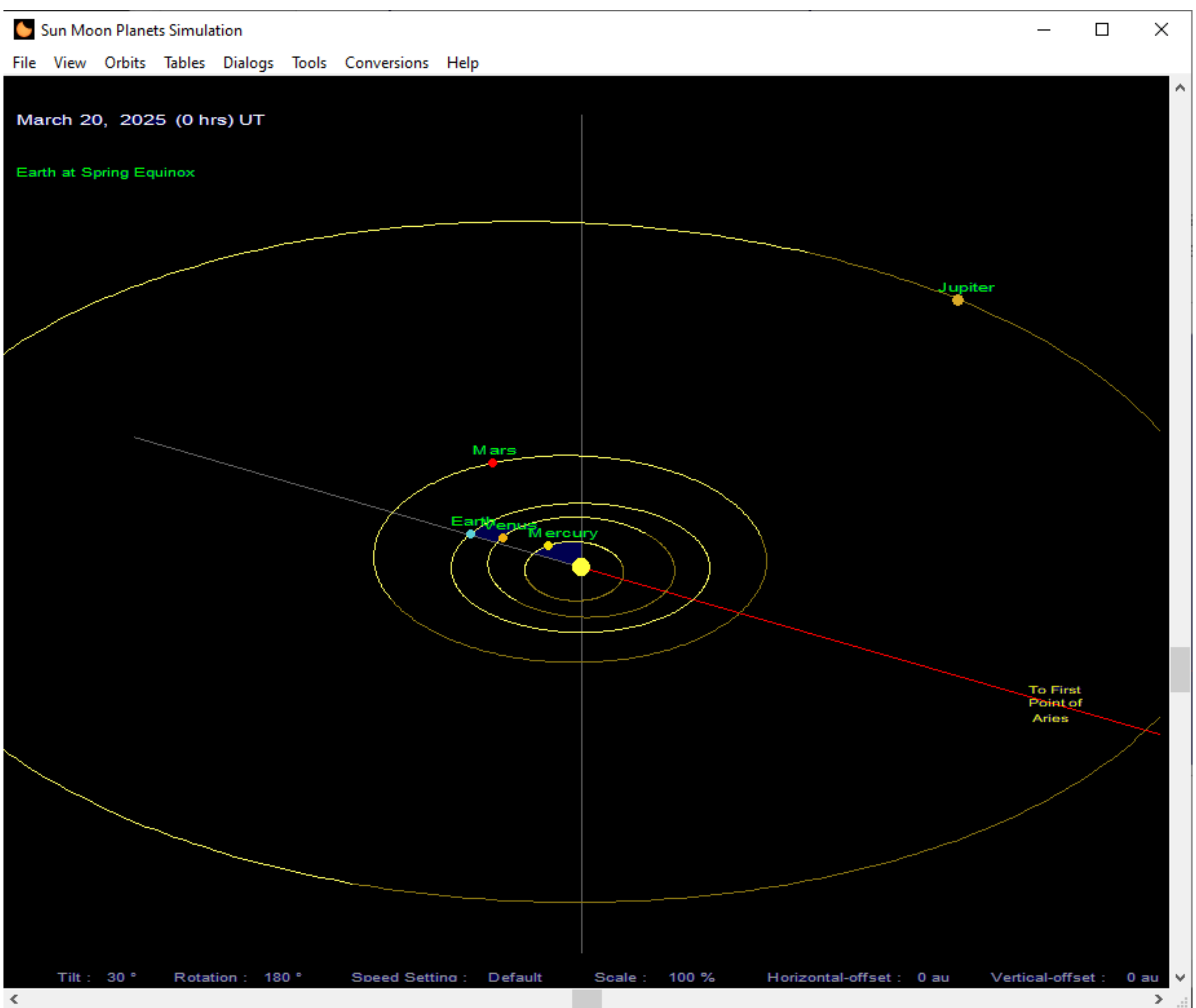
<u>Examples of Using RA and Dec in the Celestial Sphere</u>

Open the program **Sun, Moon, Planets Simulation**. On the main screen you should see the position of the planets orbiting the Sun. Also note the red line from the Sun pointing down and to the right side of the screen. That line is pointing towards the **First Point of Aries**. That is the direction for which **α**= 0h, 0m, 0s and **δ**= 0°, 0',0".

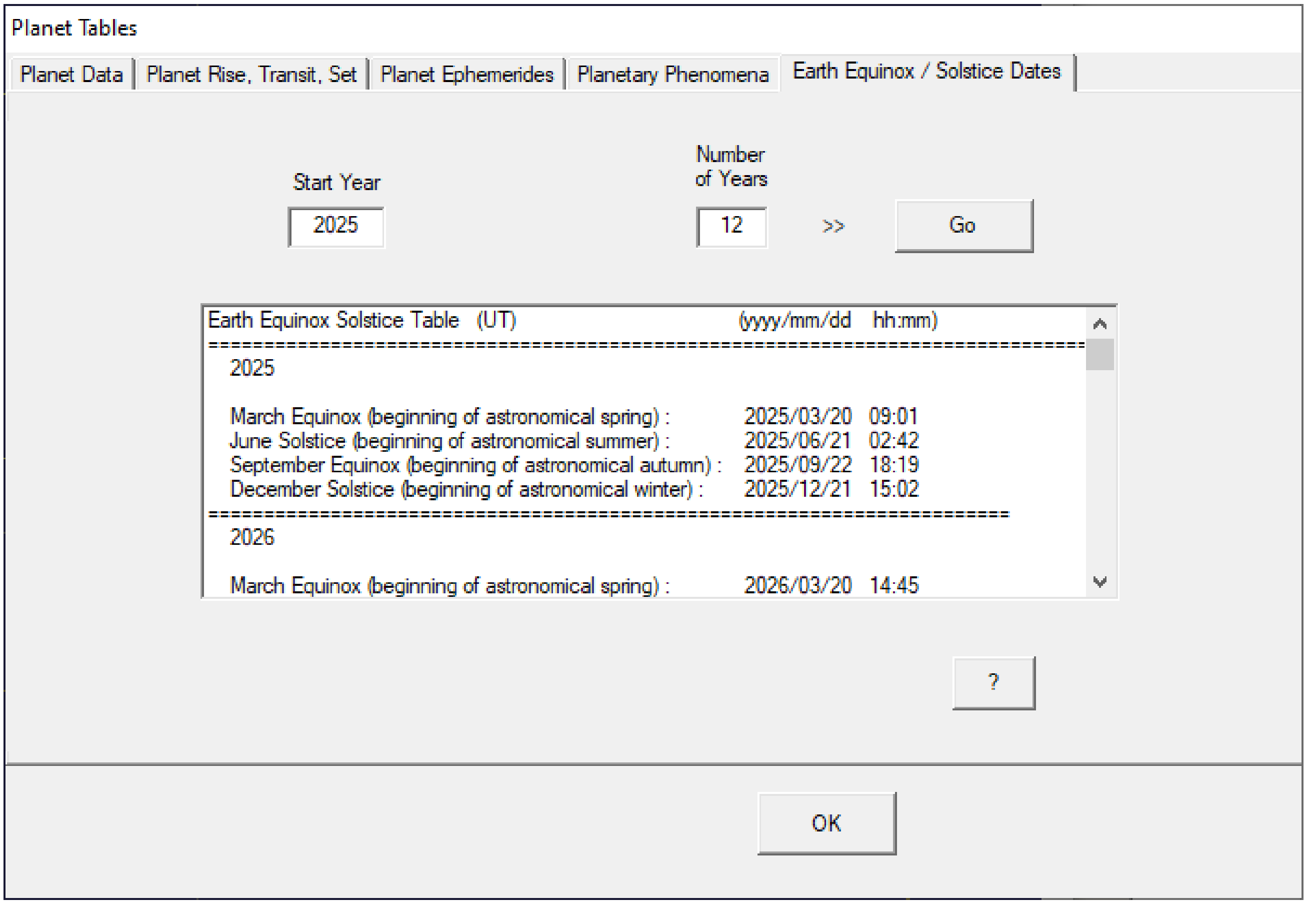
Using the **Tools** menu item, click **Select a Date**. In the dialog box, enter the date **2025, March 20** and click the button **Set/Reset Date**. Then click the **OK** button.



Look at the position of the planets on the main screen. Note in green at top left that Earth is at **Spring Equinox**. Also note that Earth is in-line with the Sun and opposite the **First Point of Aries**.

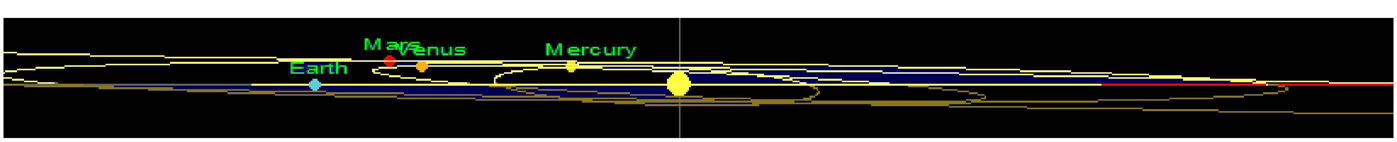


Next, select menu item **Tables** and **Planet Tables**. From that dialog box select the tab **Earth Equinox / Solstice Dates** as shown below.



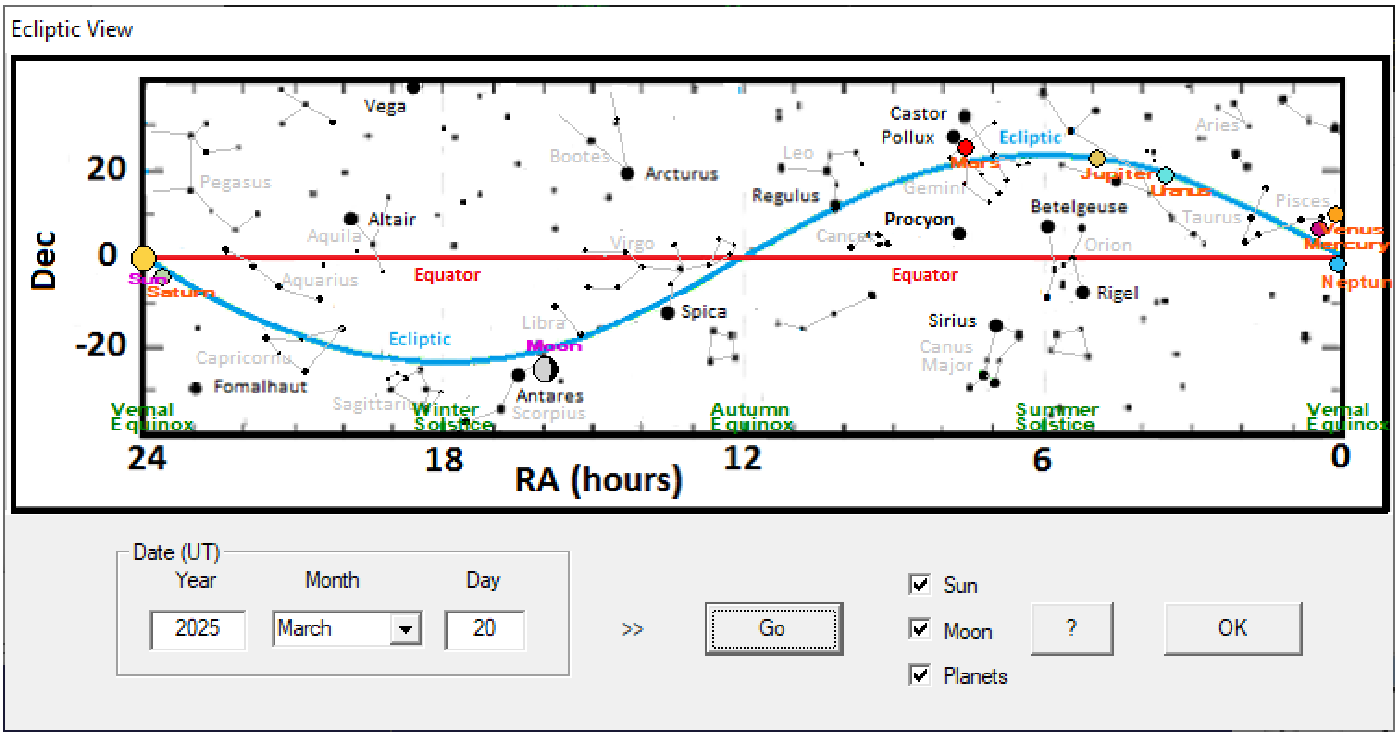
Note in the text box the date and time of the **March Equinox** - it is 2025/03/20 at 09:01. The location is for the Prime Meridian. So, at that instant of time, the Sun would be rising through the Earth's **Celestial Equator**. It is at that instant of time that Spring begins. At a different location (such as Toronto, Ontario) the first day of Spring would have begun five hours later. Toronto is five hours behind the Prime Meridian (Greenwich Mean Time, GMT) during Standard Time (EST), and four hours behind during Daylight Saving Time (EDT).

To get a different view of this event, adjust the orbit scale (**Tools || Orbit Scale**) to about 250% and then use the vertical slider to set the **Tilt** to 0°. Look closely to see the Earth and the Sun directly in line with the **First Point of Aries**.



In the flat view above, what you are looking at is edge-on of the **Ecliptic** plane. Note the line connecting the Earth and the Sun. The other planets are very close to this plane.

For another view point select menu item **Dialogs** and then **Ecliptic View**. This brings up a dialog box that shows **RA** on the horizontal axis and the **Dec** on the vertical axis. In this dialog box, set the date to 2025, March 20.

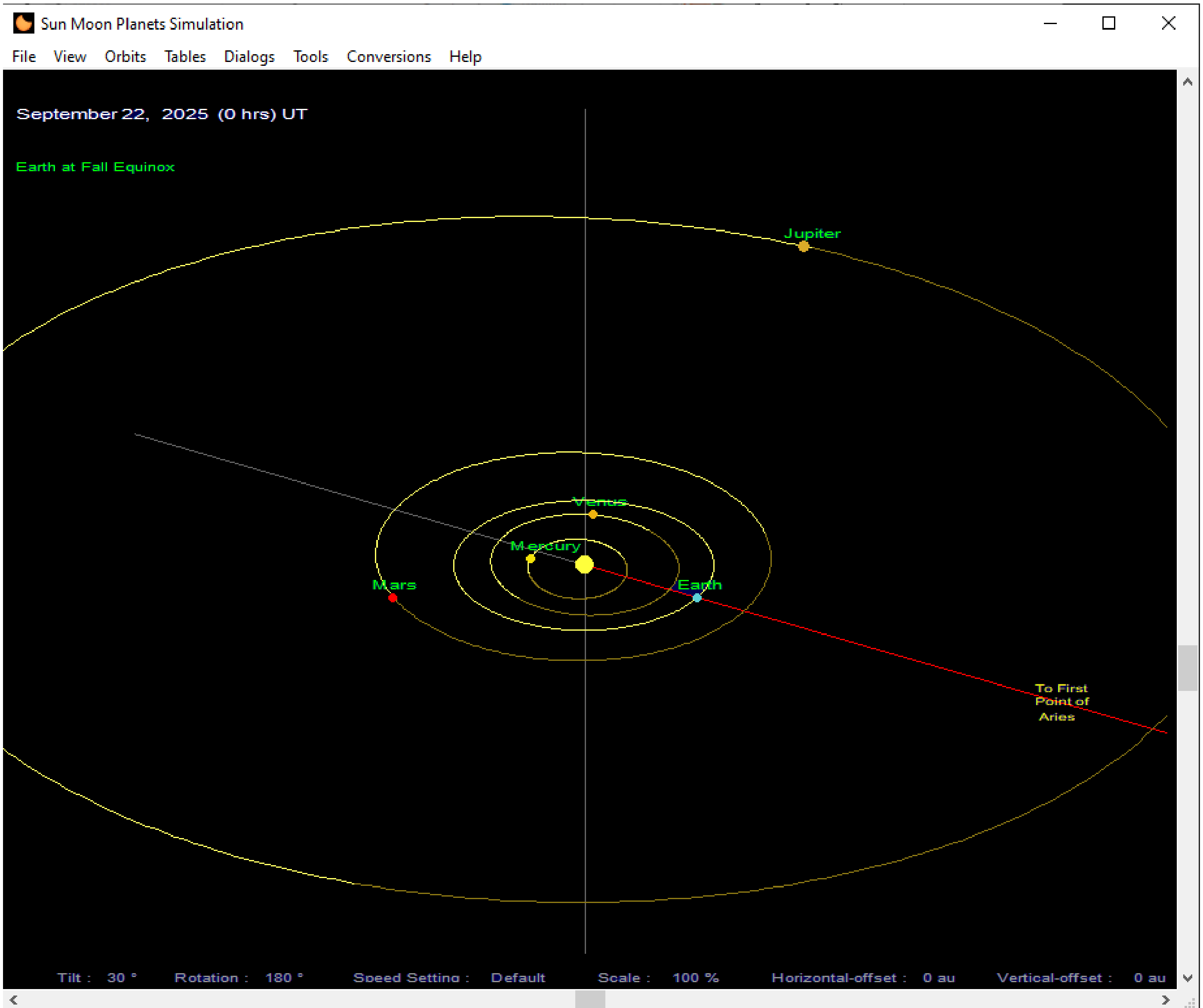


In the dialog box you can see the position of the planets, the Moon, and the Sun, all generally following along the **Ecliptic** (blue line). Note that the Sun is located at **RA** = 24 hours and **Dec** = 0°. If you add one more day the Sun appears at 0 hours (**RA**). 24 hours and 0 hours are the same. The Sun is at the **First Point of Aries**.

The **First Point of Aries** is a location; the **March Equinox**, **Spring Equinox**, **Vernal Equinox** is the date at which the Sun is at the **First Point of Aries**.

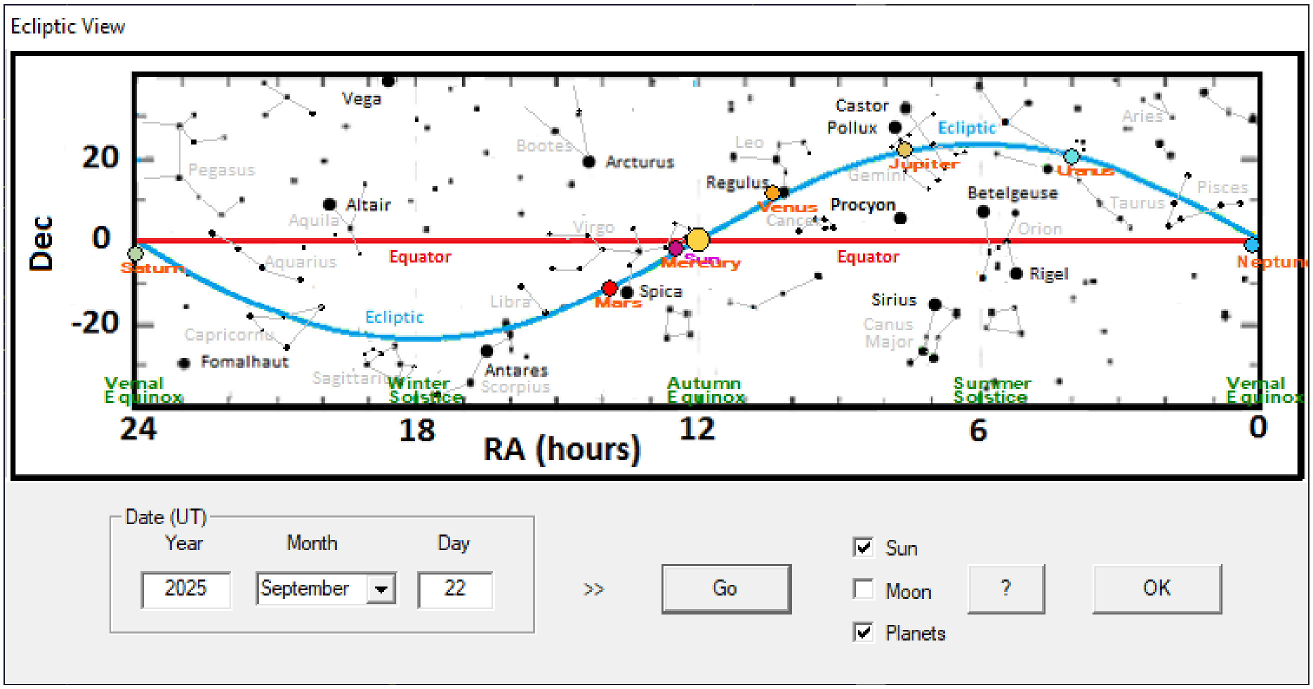
When the Sun is at 0 hours **RA** and 0° **Dec** then it is the **Spring Equinox**. Now if you go back to the main screen and change the Date to 2025, September 22 then you will see that the Earth is again in-line with the Sun but this time it is on the opposite side, between the Sun and the **First Point of Aries**.

Note in the top, left corner of that screen, the green text states the Earth is at **Fall Equinox**.

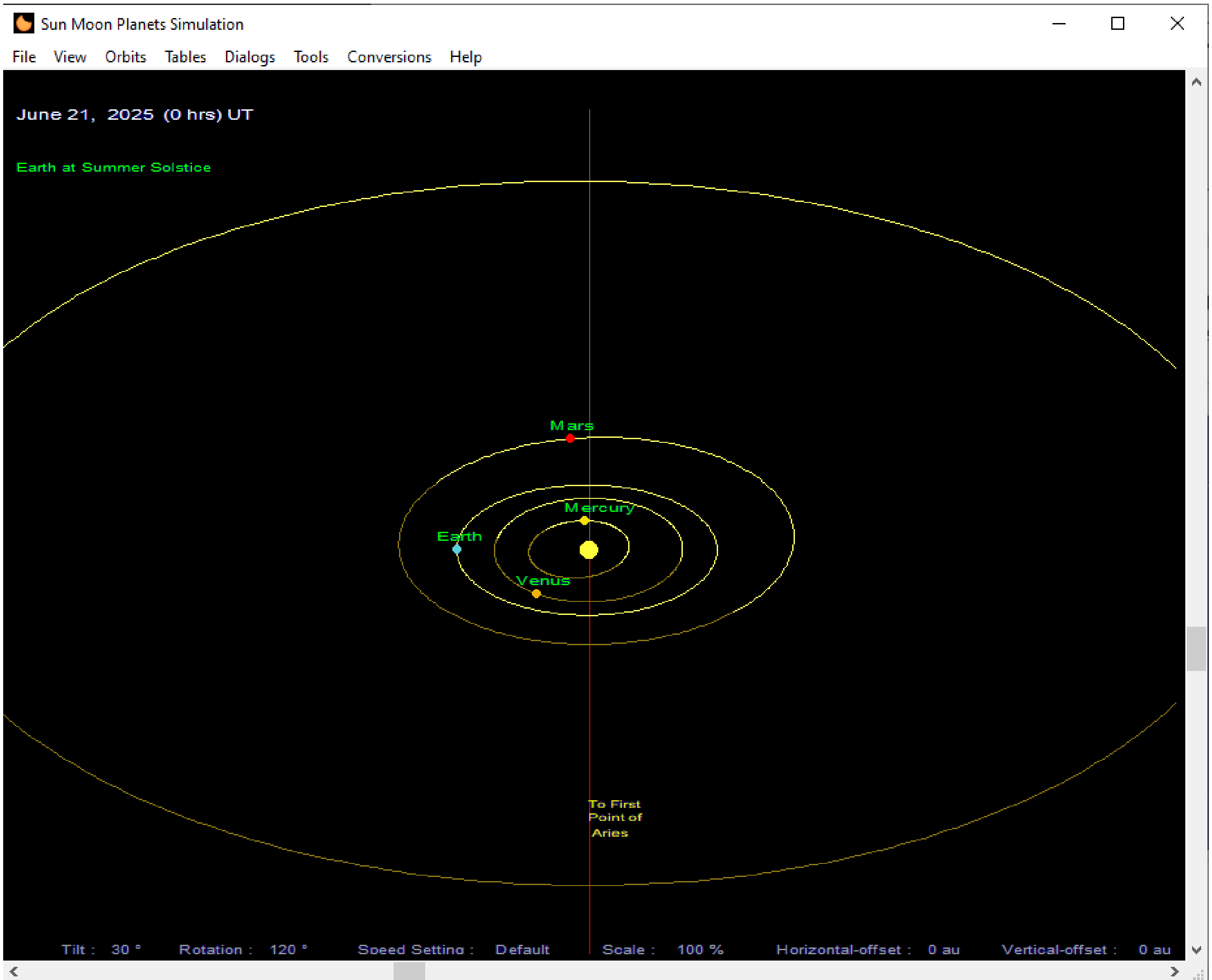


If you click menu item **Dialogs** and **Ecliptic View** and then in the dialog box set the date to 2025, September 22 (same as on the main screen). Note that the Sun is now right in the middle at RA = 12 hours and Dec = 0°.

Note, the Moon check box is switched off as the Moon is mostly blocking the Sun on that date. You will see why if you go to **Orbits || Earth Moon Sun** and set the date to Sept 22, 2025.

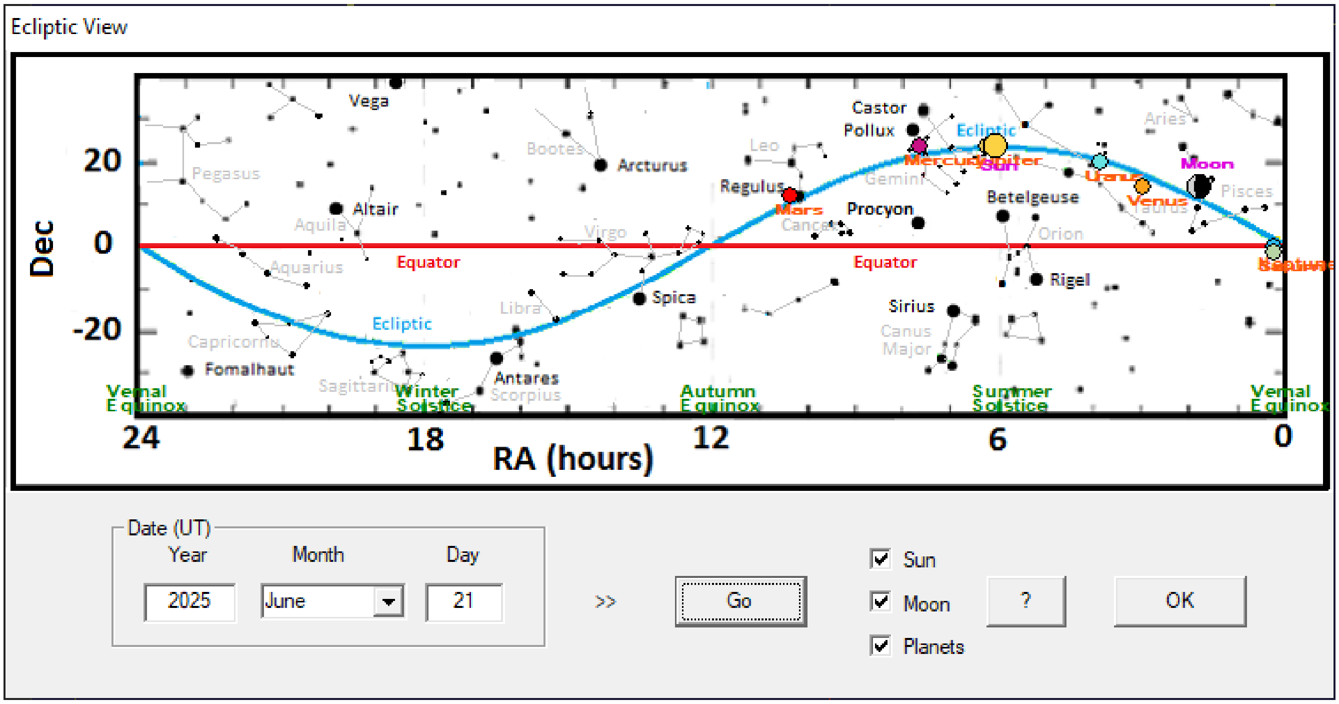


For the **Solstices**, first set the main screen date to 2025, June 21st.
On the main screen note that it is the date of the **Summer Solstice** and the position of the Earth is at 90° to the left of the Sun - or, the Sun is at 90° to the right of the Earth. Observe that the horizontal slider (rotation) has been moved to 120°.



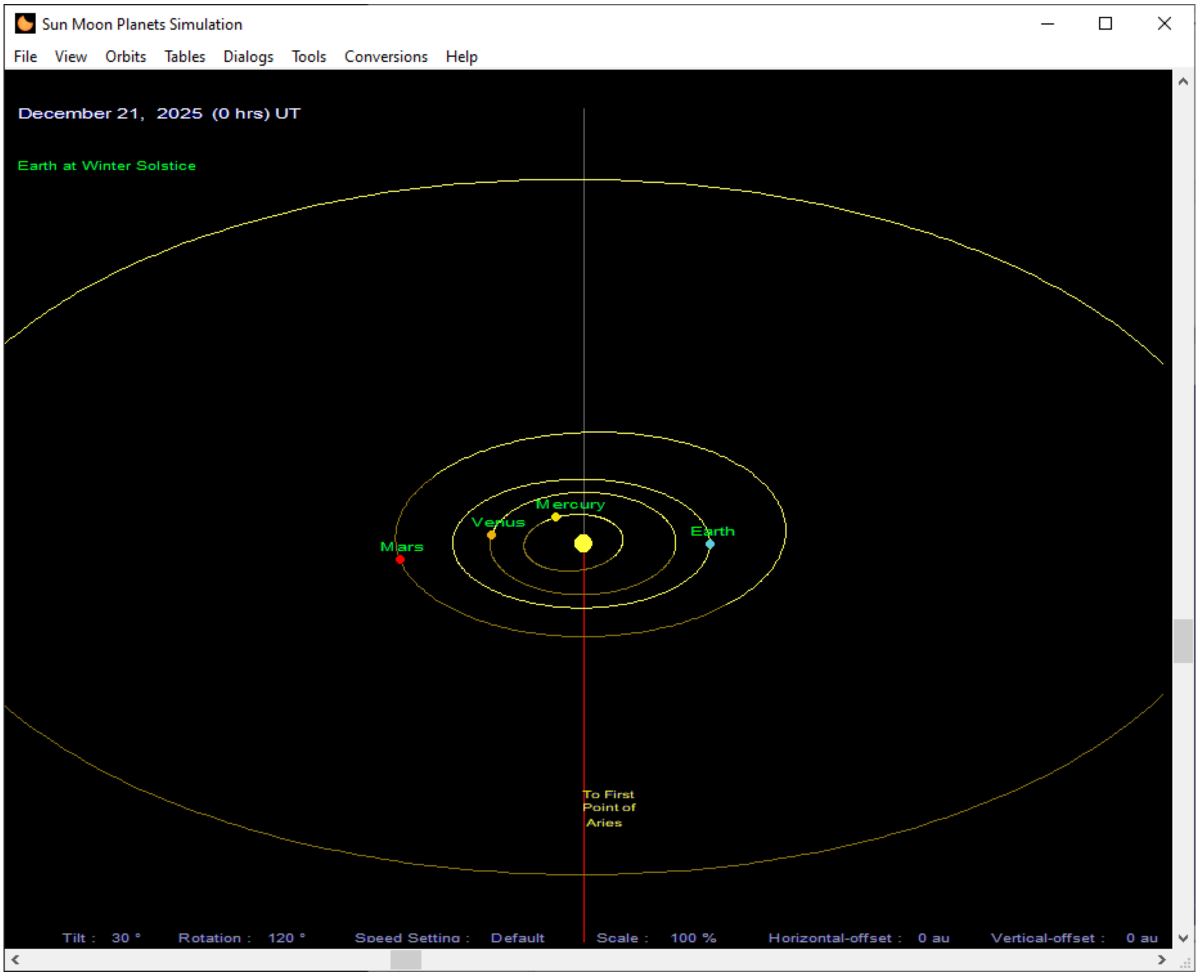
If you click menu item **Tools** and **Ecliptic View** and then in the dialog box set the date to 2025, June 21st (same as on the main screen). Note that the Sun is now at **RA** = 6 hours and **Dec** is approximately 23° (at its highest point).

Keep this 6 hours of **RA** in mind for a few minutes.

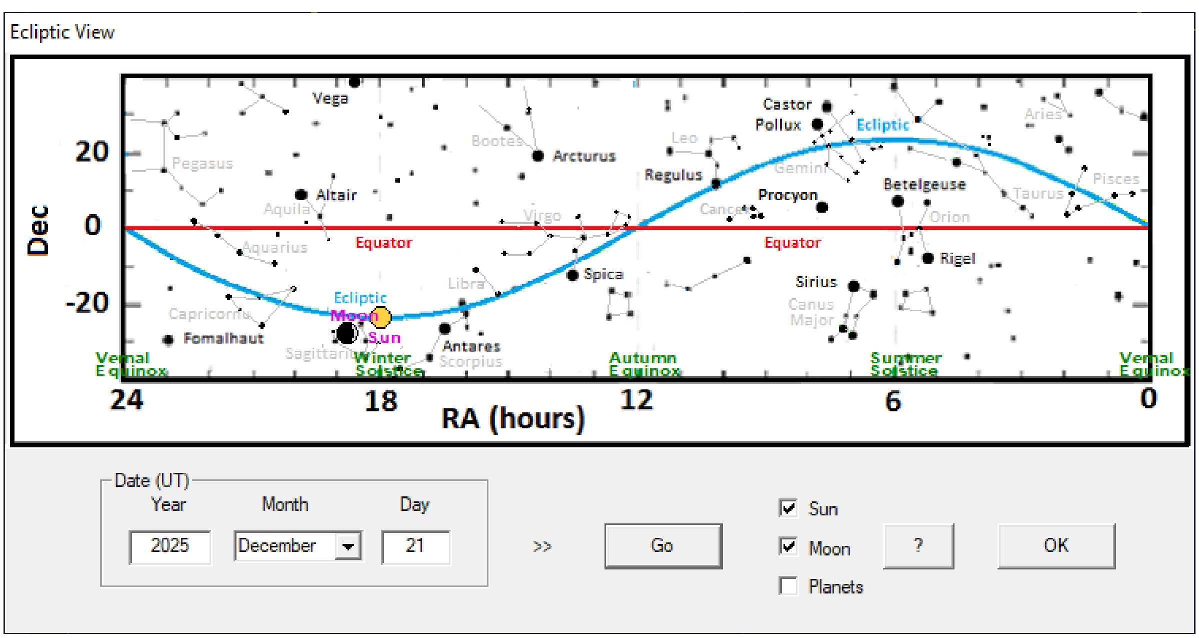


A bit of explanation here - **Right Ascension (RA)** runs from 0 hours to 24 hours similar to a clock. There are 360° degrees in a circle, same as the Earth orbiting the Sun. Therefore 24 hours corresponds to 360°. This means
1 hour of **RA** = 360° / 24 hours. Or, in other words, 1hour of **RA** corresponds to 15°. So, therefore 6 hours **RA** corresponds to 90° as is shown on the main screen.

Now, let's look at the last one, the **Winter Solstice**. Set the date on the main screen for 2025, December 21. Note the position of the Earth relative to the Sun. This time it's on the opposite side at 90° so that the Sun is to the left of the Earth.



In the **Ecliptic View**, the Sun is at 18 hours **RA**. Note the check box for Planets is unchecked; this was necessary to see the Sun at 18 hours without being blocked by the planets.



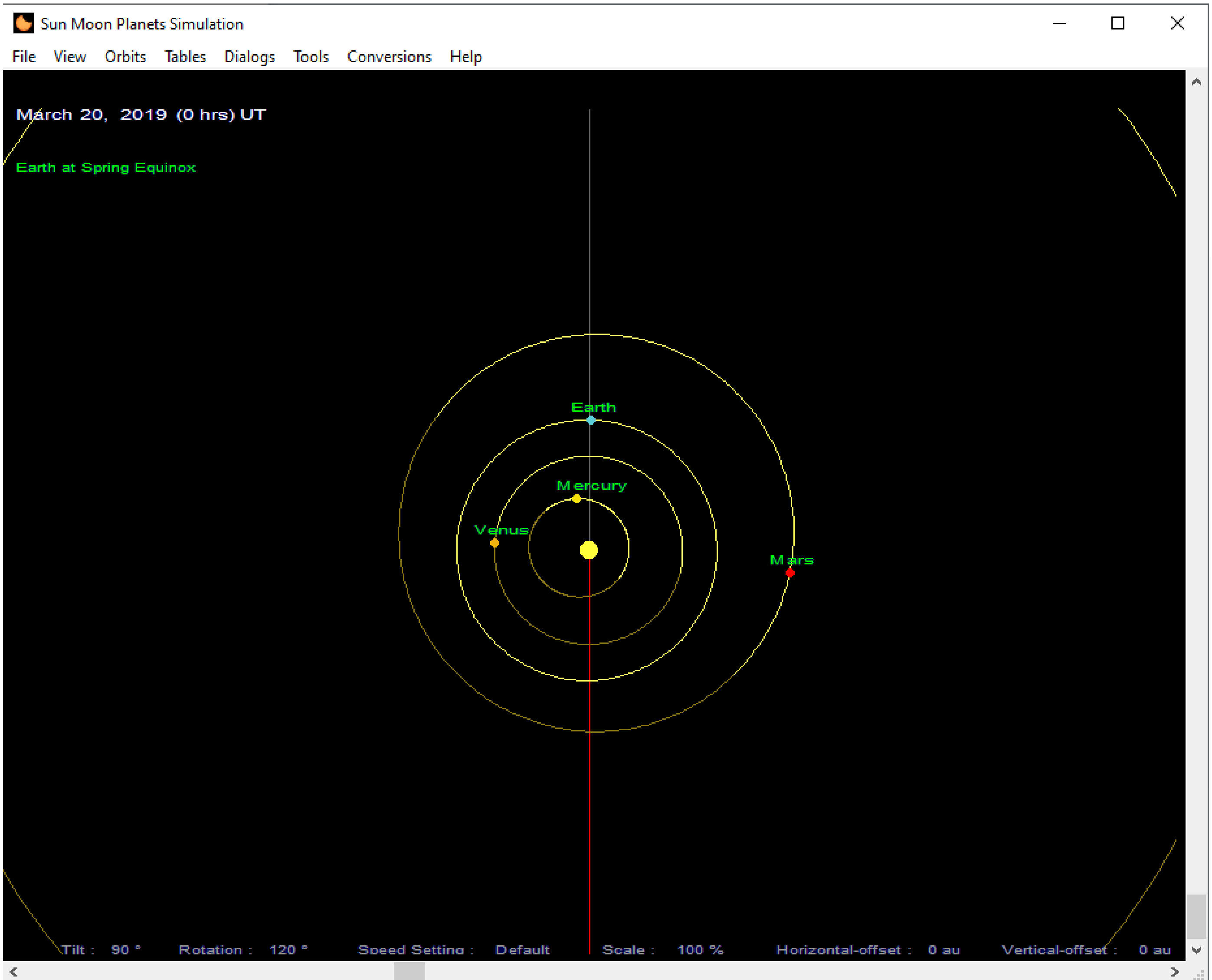
So, as the Earth revolves around the Sun throughout the year, the Sun appears to orbit the Earth and occupy a different position of **Right Ascension (RA)** and **Declination (Dec)** each day. This should not be confused with the apparent motion of the Sun as it rises and sets each day - this is due to the Earth's rotation on its axis, not on the Earths revolution around the Sun throughout the year.

To better understand this, suppose you were to pick any particular time of day (e.g. 12:00 noon) and observe the position of the Sun at the same time every day throughout the year. You would notice that the Sun appears to have slight changes in position at that same time every day. This is due to the revolution of the Earth around the Sun, not to the rotation of the Earth.

What we can take away from this is that from our position on Earth we can identify the position (**RA** and **Dec**) of any object in the sky including the Sun, Moon, planets, asteroids, comets, stars, etc. This gives every object in the sky a defined position that is not dependant on your latitude or longitude. This makes it much easier to communicate the position of an object to others without including your own particular latitude and longitude. It also allows you to standardize the position of objects such as stars that are very far away. For example, Betelgeuse (in the constellation Orion) has a **Right Ascension** of 5h 55m 10.3 s and a **Declination** of +7° 24' 25.4". It's position does not change.

But, as we have seen, celestial objects (within our solar system) are much closer and so when we state their position, it depends on the position of the Earth as it orbits around the Sun and also the position of that object as it orbits the Sun. For example: 2019, August 26, Mars is at **RA** = 10h 28m 47s and **Dec** +10° 43'24"; but later on 2019, September 26, Mars is at **RA** = 11h 42m 17s and **Dec** +3° 0' 31".

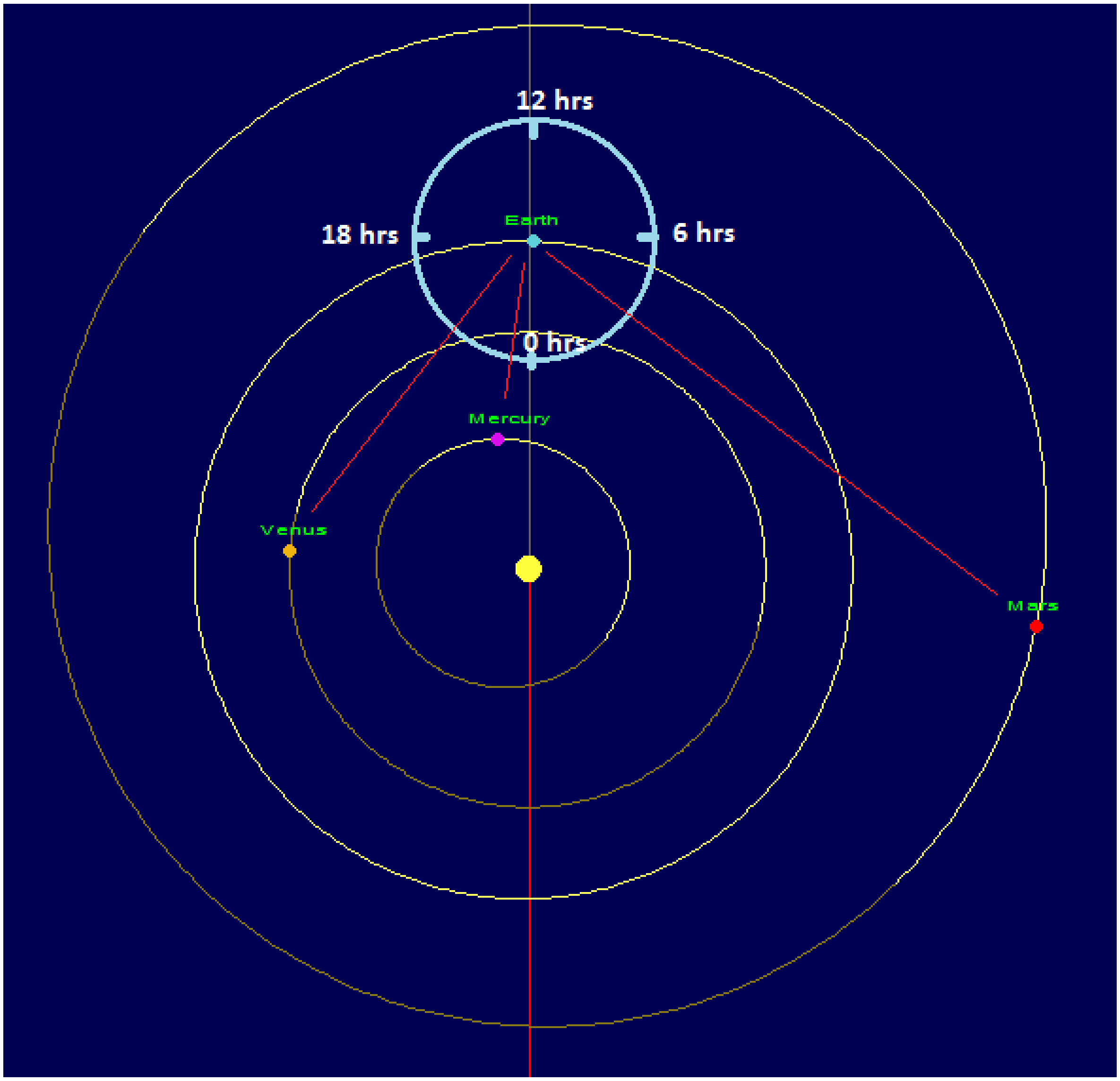
On the main screen set the date again for 2019, March 20 (although the following exercise can be done for any date). Next, move the vertical slider to the bottom so that **Tilt** is at 90° and then the horizontal slider so that **Rotation** is 120°. Here is what the main screen should look like.



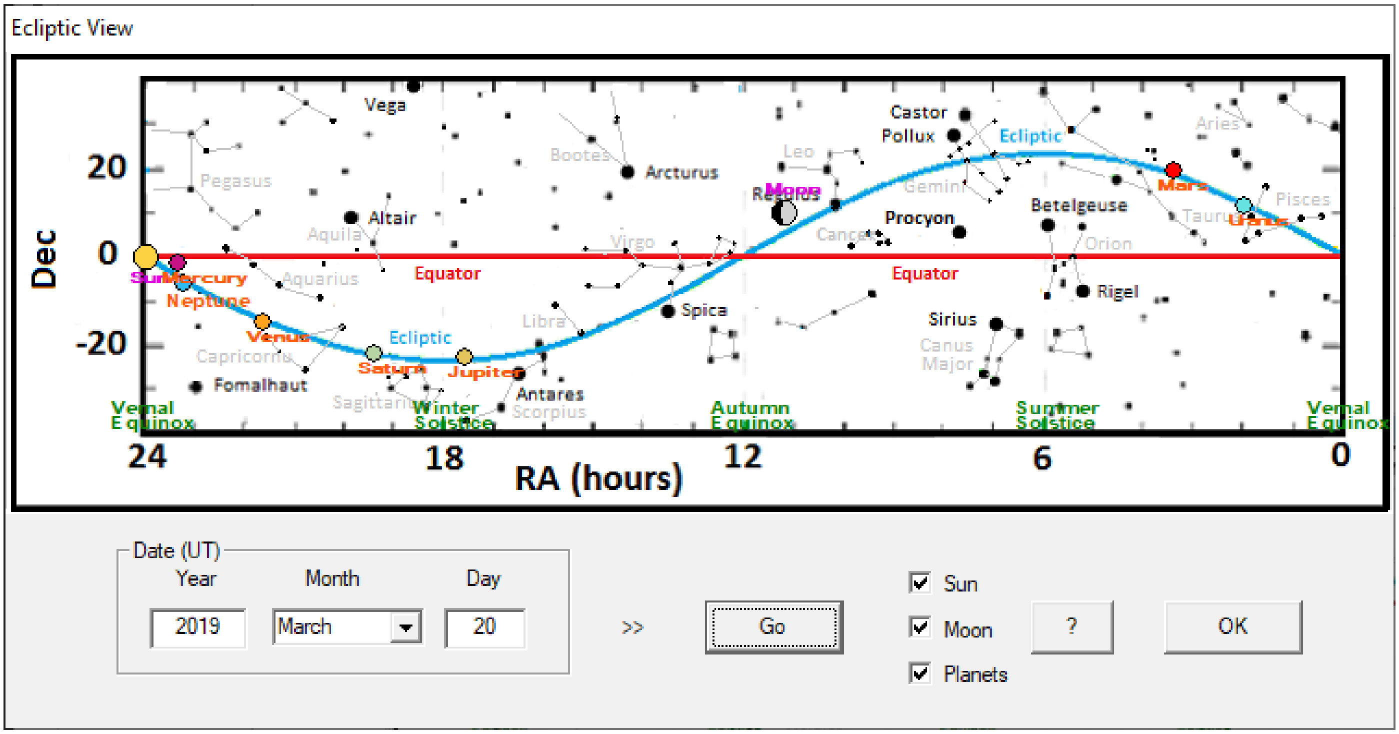
Note, the red line pointing to the **First Point of Aries** is straight down.

Now, imagine a 24 hour **Right Ascension** clock around the Earth as shown below. The circle around Earth is marked off at 0 hours (at the bottom), 6 hours, 12 hours (at the top), 18 hours, and back to 0 hours.

On this date, from the view at Earth, the Sun is at 0 hours. The angular position of Mars appears about halfway between 0 hours and 6 hours, so a quick estimate might be around 3 hours. A quick estimate of Venus might be around 21 to 22 hours and for Mercury around 23 hours. If you go to the **Planet Tables** and check the **Planet Ephemerides** for Mars, Venus, and Mercury for 2019, March 20, you should see that those approximations are reasonably accurate.



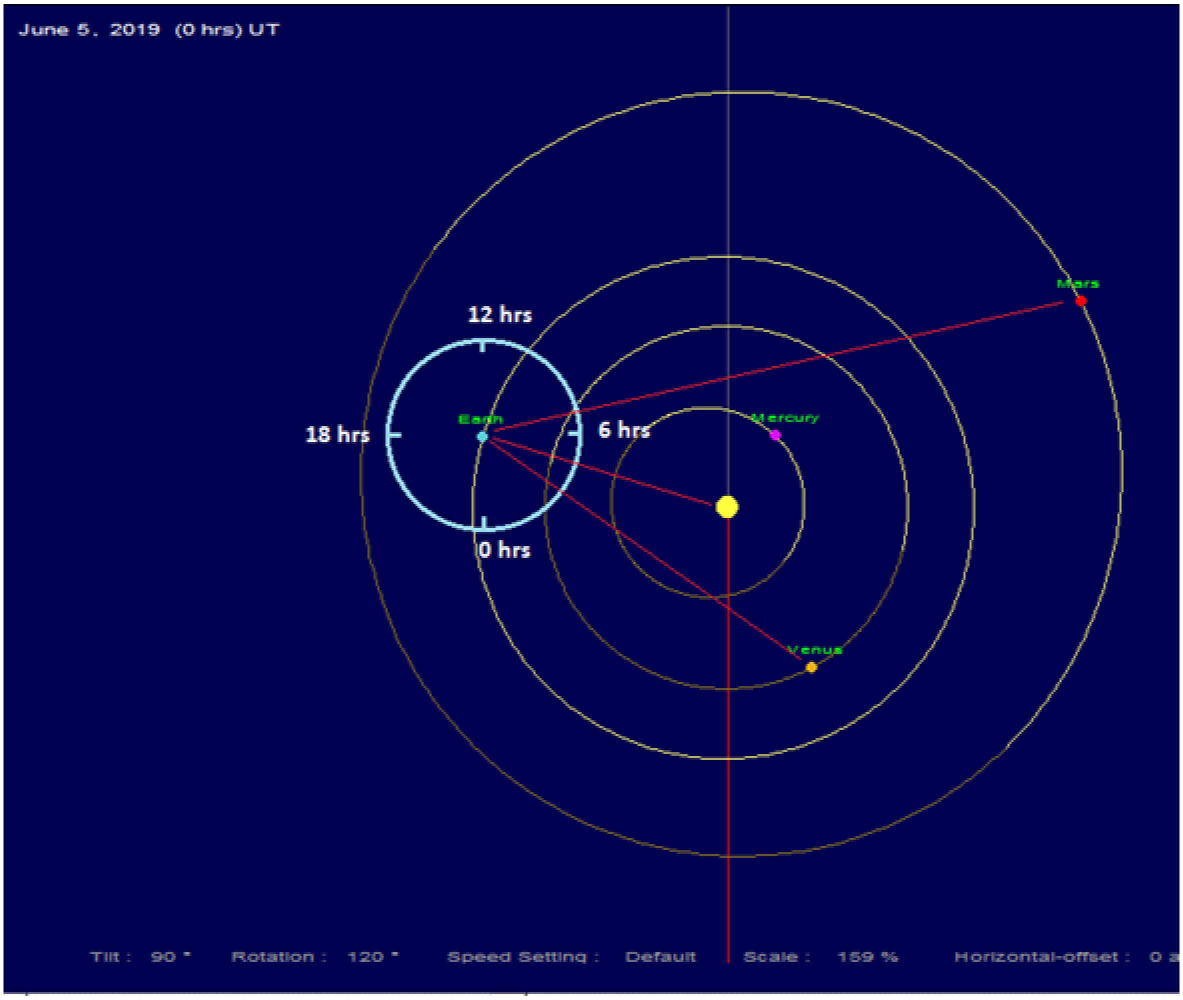
Also, if you bring up the dialog box for the **Ecliptic View**, set the date for 2019, March 20 and then look for the **RA** and Dec of those three planets. Again, Mars is around 3 hours **RA**, Venus is around 21 to 22 hours and Mercury is about 23 hours.



Any time you look at the main screen, note the position of Earth and the other planets. Imagine the 24 hour **Right Ascension** clock with 0 hours straight down and 12 hours straight up. Then try to guess at the RA of the planets relative to the Earth. To check your guess just go to the **Planet Tables** and select the **Ephemerides** tab, set the date and select the planets. Compare your approximations of **RA** with those in the table.

Try any different date - for example, set it to 2019, June 5. Make sure the red line for the **First Point of Aries** is pointing straight down.

Check the figure below. In this position, the view from Earth, the Sun appears to be around 5 hours (**RA**). The angular position of Mars appears to be about 7 hours. A quick estimate of Venus might be between 3 and 4 hours and for Mercury around 6 hours. If you go to the **Planet Tables** and check the **Planet Ephemerides** for Mars, Venus, and Mercury for 2019, June 5 you should see that those approximations are reasonably accurate. Also, the **Ephemerides** in the **Sun Tables** (set the date) shows the Sun to be 4h 50m 55s, very close to the estimated 5 hours.



If you bring up the **Ecliptic View** and make sure the date is the same, 2019, June 5, you should note the Sun appears to be around 5 hours, Venus is close to 4 hours, Mars at 7 hours, and Mercury at 6 hours.

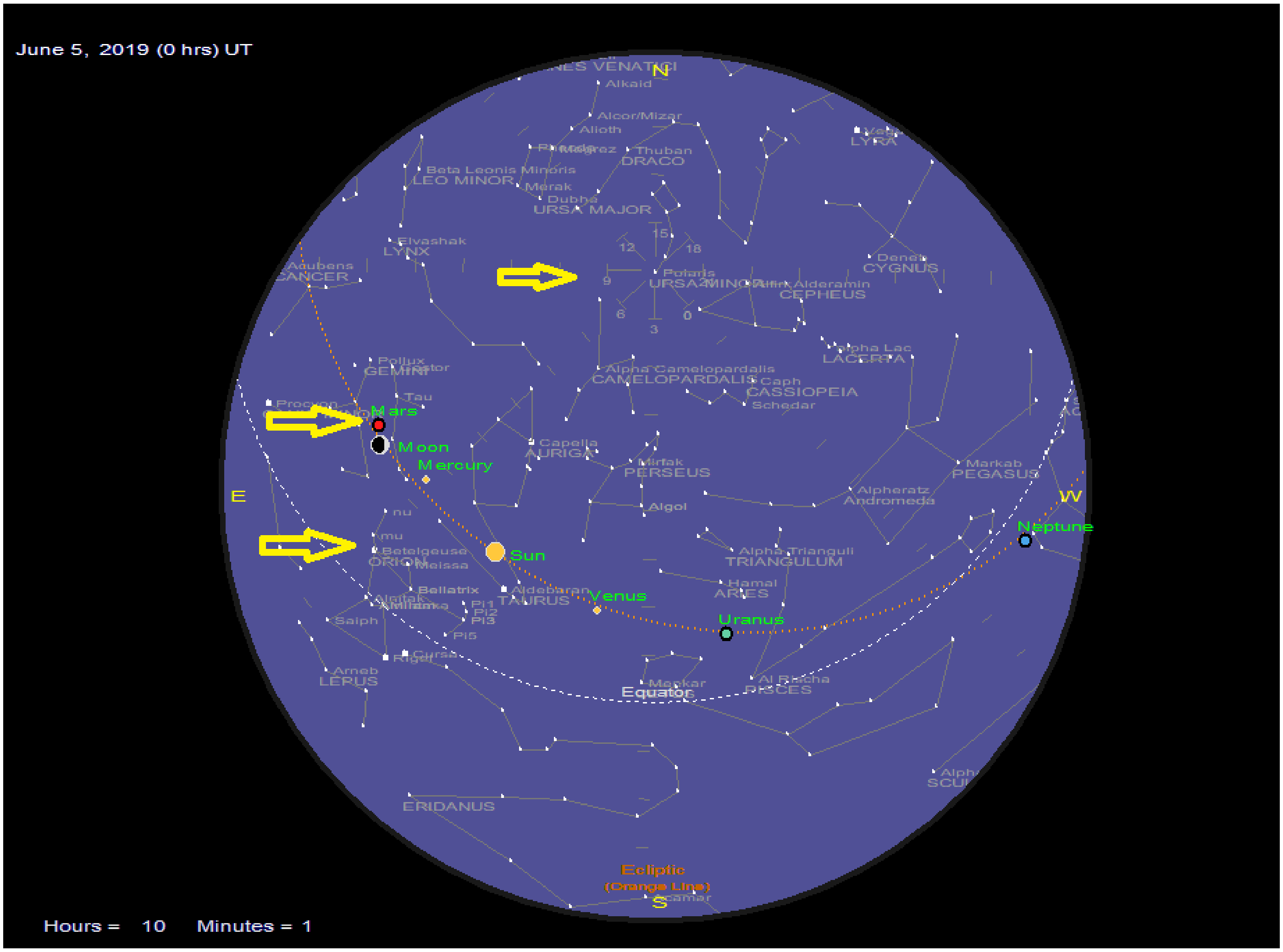
So, using the **Sun Moon Planet Simulation** allows you to very quickly get an estimate of the position (**RA**) of the Sun, Moon, or planets on any date. Just move the red line straight down and imagine a **Right Ascension** clock around Earth.

When you are outside observing the night sky, if you know the fixed **RA** of a well-known star, and the current **RA** of a planet, you can use the star to quickly find the planet. Keep in mind that 1 hour of **RA** is equal to 15°.

For example, Betelgeuse (in the constellation Orion) is a well-known star, easy to find, and has a **Right Ascension** of approximately 6 hours and is a bit below the **Ecliptic**. On 2019, June 5, we find in this program that Mars is positioned around 7 hours. If you went outside at night on that date, found Betelgeuse, looked up a bit and about 15° to the left (east), there was Mars.

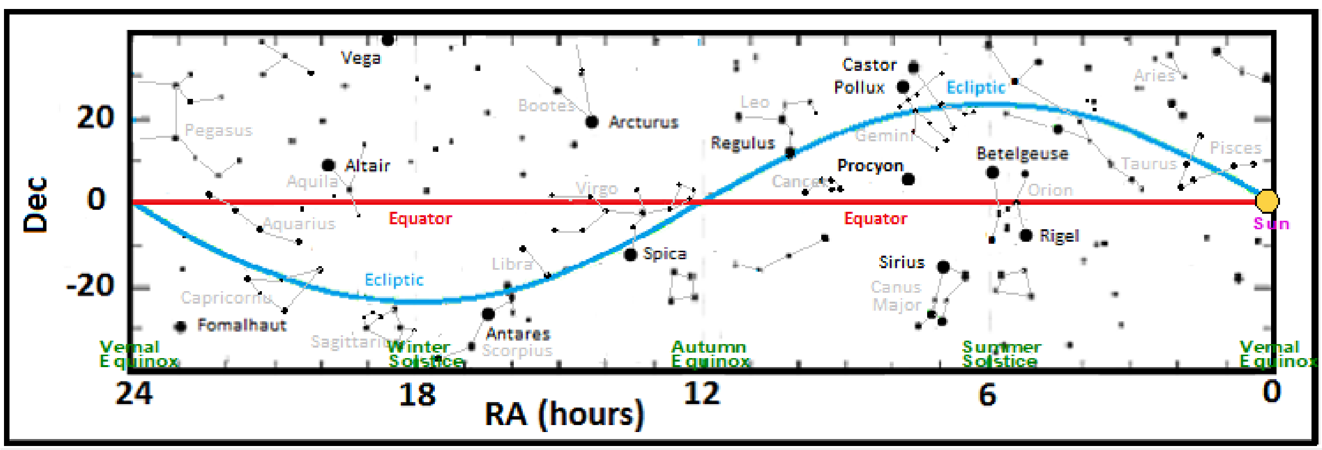
On June 5, 2019, the difference between Mars at 7 hours and Betelgeuse at 6 hours is 1 hour. Recall that 1 hour of **Right Ascension** is equivalent to 15°.

You can verify this observation right in **Sun Moon Planets Simulation**. Go to menu item **Orbits || Sky View with Orbiting Planets**. Make sure the date is set to 2019, June 5. Adjust the slider along the bottom of the window to about 10 hours. Now, find the star Polaris (North Star) and note that an **RA** clock surrounds it. Follow the clock line marked 6 hours down below the ecliptic to find Betelgeuse. Then back up to the ecliptic line (orange) and to the East, you should see Mars – at about 7 hours according to the clock around Polaris. Finally, if you go to the **Planet Tables**, **Ephemeris** you should see Mars at 6 h 56m for June 5, 2025.



<u>Precession</u>

Refer to the **First Point of Aries** at **RA** = 0 hours and **Dec** = 0° (see **Vernal Equinox** on the right-side below). At this point the **Ecliptic** (blue line) and the **Celestial Equator** (red line) intersect. As shown below, **Right Ascension** increases east (left) of that point. This is the point where the Sun appears to cross the **Celestial Equator** going north **(+ Dec**.) and the length of day and night are equal. This is the date of the **Vernal Equinox**.



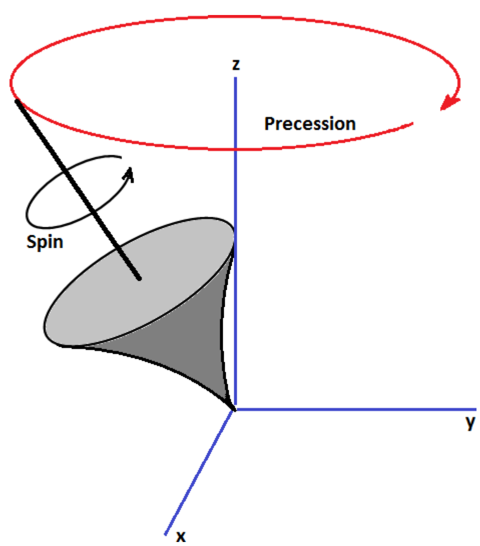
To see this dialog box, click menu item **Dialogs** and select **Ecliptic View**.

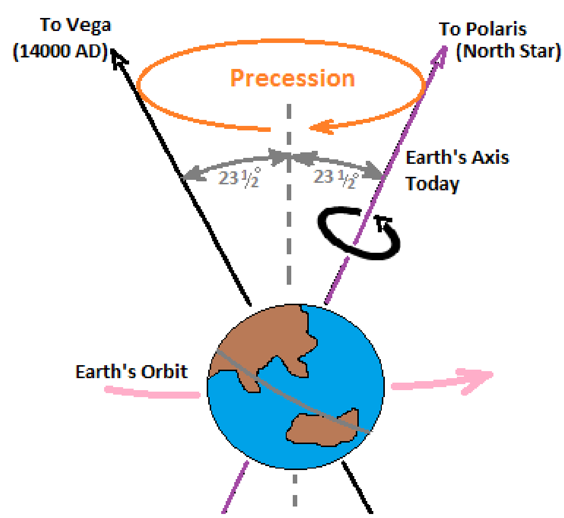
Why is the **First Point of Aries** now located in the constellation **Pisces**? Observe in the dialog box the location of **RA** = 0 hours and **Dec** = 0°. You should see the constellation **Pisces** (light gray letters) near that point. If you look carefully in the dialog box, **Aries** is a bit above **Pisces** and east (left) of it. The **First Point of Aries** used to be in the constellation of **Aries** but has moved westward (to the right) towards the constellation **Pisces**. The **First Point of Aries (vernal equinox)** was in the constellation **Aries** about 2,000 years ago and slowly moved into the constellation **Pisces**. The change was caused by Earth's axial wobble (precession), putting it now in **Pisces**.

Precession can be described as the wobble in the rotational axis of a rotating body (such as a top, or the Earth). Earth's precession has resulted in the equinoxes slowly moving westward relative to the background stars. In other words, the **Ecliptic** itself is moving westward. Precession is a result of the Earth's rotation and the gravitational pull of the planets, the Moon and the Sun. The Moon and the Sun are by far the most significant cause of precession.

The rate of precession is about 1 degree every 71.6 years (or 50.3 seconds of arc per year). In 14000 AD the Earth's axis will be pointing at Vega. For the Earth to complete a full cycle of 360° will take nearly 26000 years. At that time Polaris would be the North Star again.

The point at where the Sun crosses the **Celestial Equator** heading north along the **Ecliptic** is in the direction of the **First Point of Aries**. In 150 B.C. when Ptolemy (*Almagest)* mapped the constellations, **Aries** was in that position.





Appendix

Python Apps:
To use these python scripts, copy them and save them using **Notebook**. Save them as **filename.py**. Then, at the Windows Command Prompt (**CMD**), type **python filename.py** (Make sure filename.py is in the proper directory otherwise it won't run – you might want to save the filename.py to your **Desktop** and then when you open Windows Command Prompt (**CMD**), change the directory from **C:\Users\Owner** to **C:\Users\Owner\Desktop** - this is done by using the command **cd Desktop** , then at the prompt type **python filename.py**)

A. Converting Julian Day Number

Before running this script, you must install astropy.
At the Windows Command Prompt (CMD) type **pip install astropy**

```
from astropy.time import Time
import datetime

# From Gregorian to JD
dt = datetime.datetime(2025, 12, 25, 0, 0, 0)
t = Time(dt, scale='utc')
print(f"JD using astropy: {t.jd}") # Output: 2461034.5

# From JD to Gregorian
t_jd = Time(2461034.5, format='jd', scale='utc')
print(f"Date using astropy: {t_jd.datetime}")
```

B. Converting Degrees and Radians
(reference: google.com, search converting degrees and radians python)

```
import math
#Degrees to Radians: Use math.radians().
degree_value = 90
radian_value = math.radians(degree_value)
print(radian_value)
# Output: 1.5707963267948966 (which is approximately π/2)
#Radians to Degrees: Use math.degrees().
radian_value = math.pi # Use the constant for pi
degree_value = math.degrees(radian_value)
print(degree_value)
# Output: 180.0
```

C. Newton-Raphson Method for Solving Keplers Equation
(reference: google.com, search solve keplers equation python)

```
import numpy as np

def solve_kepler(M, e, tolerance=1e-10, max_iter=100):
    """Solves Kepler's Equation (M = E - e*sin(E)) for E using Newton's
method."""
    # Initial guess for E (often M is a good start, or use Machin's
approximation for better speed)
    E = M + e * np.sin(M) # A simple starting point

    for _ in range(max_iter):
        f_E = E - e * np.sin(E) - M # The function we want to be zero
        f_prime_E = 1 - e * np.cos(E) # The derivative of f_E

        delta_E = f_E / f_prime_E # Newton-Raphson step
        E -= delta_E
```

```
        if abs(delta_E) < tolerance:
            return E # Converged

    # If it doesn't converge within max_iter
    print("Warning: Kepler solver did not converge.")
    return E

# --- Example Usage ---
mean_anomaly_rad = np.radians(231.53975) # M in radians (e.g., 100
degrees)
eccentricity = 0.0795753  # e for the orbit

eccentric_anomaly_rad = solve_kepler(mean_anomaly_rad,
eccentricity)
print(f"Mean Anomaly (M): {np.degrees(mean_anomaly_rad):.2f}
deg")
print(f"Eccentricity (e): {eccentricity}")
print(f"Eccentric Anomaly (E): {np.degrees(eccentric_anomaly_rad):.2f}
deg")

# Verify (M should be close to E - e*sin(E))
calculated_M = eccentric_anomaly_rad - eccentricity *
np.sin(eccentric_anomaly_rad)
print(f"Verification (Calculated M): {np.degrees(calculated_M):.2f}
deg")
```